GCE EXAMINERS' REPORTS

GCE
FURTHER MATHEMATICS
AS/Advanced

SUMMER 2023

Introduction

Our Principal examiners' reports offer valuable feedback on the recent assessment series. They are written by our Principal Examiners and Principal Moderators after the completion of marking and moderation, and detail how candidates have performed.

This report offers an overall summary of candidates' performance, including the assessment objectives/skills/topics/themes being tested, and highlights the characteristics of successful performance and where performance could be improved. It goes on to look in detail at each question/section of each unit, pinpointing aspects that proved challenging to some candidates and suggesting some reasons as to why that might be.[i]

The information found in this report can provide invaluable insight for practitioners to support their teaching and learning activity. We would also encourage practitioners to share this document – in its entirety or in part – with their learners to help with exam preparation, to understand how to avoid pitfalls and to add to their revision toolbox.

Further support

Document	Description	Link
Professional Learning / CPD	WJEC offers an extensive annual programme of online and face-to-face Professional Learning events. Access interactive feedback, review example candidate responses, gain practical ideas for the classroom and put questions to our dedicated team by registering for one of our events here.	https://www.wjec.co.uk/home/professional-learning/
Past papers	Access the bank of past papers for this qualification, including the most recent assessments. Please note that we do not make past papers available on the public website until 6 months after the examination.	www.wjecservices.co.uk or on the WJEC subject page
Grade boundary information	Grade boundaries are the minimum number of marks needed to achieve each grade. For unitised specifications grade boundaries are expressed on a Uniform Mark Scale (UMS). UMS grade boundaries remain the same every year as the range of UMS mark percentages allocated to a particular grade does not change. UMS grade boundaries are published at overall subject and unit level. For linear specifications, a single grade is awarded for the overall subject, rather than for each unit that contributes towards the overall grade. Grade boundaries are published on results day.	For unitised specifications click here: Results, Grade Boundaries and PRS (wjec.co.uk)

Exam Results Analysis	WJEC provides information to examination centres via the WJEC secure website. This is restricted to centre staff only. Access is granted to centre staff by the Examinations Officer at the centre.	www.wjecservices.co.uk
Classroom Resources	Access our extensive range of FREE classroom resources, including blended learning materials, exam walk-throughs and knowledge organisers to support teaching and learning.	https://resources.wjec.co.uk/
Bank of Professional Learning materials	Access our bank of Professional Learning materials from previous events from our secure website and additional pre-recorded materials available in the public domain.	www.wjecservices.co.uk or on the WJEC subject page.
Become an examiner with WJEC.	We are always looking to recruit new examiners or moderators. These opportunities can provide you with invaluable insight into the assessment process, enhance your skill set, increase your understanding of your subject and inform your teaching.	Become an Examiner | WJEC

Subject Officer's Executive Summary

The Summer 2023 GCE Further Mathematics AS and A2 papers were generally accessible to all candidates, with an appropriate balance of differentiating questions to challenge the most able candidates.

Overall, candidates performed better in the Summer 2023 Further Pure Mathematics and Further Statistics papers, relative to the Summer 2022 series. However, poor algebraic skills, coupled with careless arithmetic errors, often cost candidates valuable marks.

Other areas for improvement are as follows:

Areas for improvement	Classroom resources	Brief description of resource
Pre-requisite knowledge from GCE Mathematics Unit 1 and Unit 3	Past papers, including legacy papers	Questions to help consolidate and strengthen GCE Mathematics Unit 1 and Unit 3 skills, e.g. algebraic manipulation.
Exponential Distribution	WJEC Resource, available here.	Notes, worked examples, and questions with worked solutions.
Vertical Circles	Past papers, including legacy papers	The topic of vertical circles is inherited from the legacy specification, so legacy past papers and resources can provide a bank of questions.

FURTHER MATHEMATICS

General Certificate of Education

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FURTHER PURE MATHEMATICS A – AS UNIT 1

Overview of the Unit

The candidates performed very well on a high number of occasions and there were some excellent scripts, including some scoring all 70 marks. However, as in previous series, poor algebraic skills were apparent and often proved costly. Candidates often performed well on the beginning and middle sections of the paper, finding the final three questions more challenging.

Comments on individual questions/sections

Q.1 This question was well answered by many candidates. However, some candidates incorrectly dealt with $(\lambda i)^2$ leading to a linear equation to solve, whilst others failed to note the information in the question that λ was a positive constant.

Q.2 Part (a) was answered well by the majority of candidates, with the most common errors being sign errors. Part (b) was answered well by many candidates, particularly those candidates who heeded the 'hence' in the question to use the inverse matrix from part (a). However, some candidates did not heed the 'hence' and found three pairs of simultaneous equations to solve. This method, if used correctly, enabled candidates to gain full credit.

Q.3 Part (a) was answered very well by candidates, with the vast majority noting the complex conjugate as another root of the equation. Part (b) was answered in a variety of ways by candidates. Some candidates used the two known roots to form a quadratic factor, to rewrite the quartic into two quadratic factors, and then solved the second quadratic equation – these candidates usually scored full marks, but careless sign errors were seen on occasions, leading to incorrect complex roots for this second quadratic equation. Some candidates used the expressions for roots of polynomials and usually scored full marks. However, some candidates decided to expand $(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$, to equate the coefficients with those given in the question – this method could lead to full credit, but errors in the expansion were seen regularly.

Q.4 Part (a) was well answered by the majority of candidates, although some candidates made errors by multiplying the matrices in the wrong order. In part (b), although candidates were able to formulate two linear equations, they were often unable to articulate how many invariant points existed under the transformation, but instead gave a description of the points.

Q.5 Overall, a well-answered question.

Q.6 Candidates seemed very familiar with the method required to answer this question, but the coefficient of 2 on the right-hand side caused some issues, e.g. if candidates chose to square both sides of the equation, they would forget to square the '2'. Whilst this error would still lead to the equation of a circle, accuracy marks were lost.

Q.7 Most candidates were able to gain at least the first three marks of this question by working out the result for $n = 1$, and then considering $n = k$ and $n = k + 1$. When calculating the entries of the $n = k + 1$ matrix, the top-right entry proved problematic. Some candidates simply noted the $(k + 1)$ form, as they knew the form of the target matrix. Others had poor algebraic skills and, through a series of errors, arrived at the required entry, such as $2 \times 2^k = 4^k$ and $5 \times 2^k = 10^k$ only to factorise 2 and 5 to return to 2^k.

Q.8 Candidates often started this question well, but were frequently unable to correctly manipulate their algebraic expressions, e.g. multiplying and simplifying fractions, for the coefficients of the new cubic equation, into a form that would allow them to make use of the results obtained earlier in the question.

Q.9 Parts (a) and (b) were generally well answered by many candidates. However, part (c) proved challenging for many candidates. Of those who made use of the formula in the Formula Booklet, many used the (x, y)-coordinates, rather than the corresponding coordinates in the (u, v)-plane.

Q.10 This was the most difficult question on the paper for many candidates. Few candidates correctly interpreted the information given in the question that the series ended with an odd number. Many candidates simply noted $\sum r^3 - \sum (r + 1)^3$ with identical ranges, or something similar, and very few marks, if any, could be awarded. However, it was pleasing to see some excellent responses to the question, with the three methods noted in the mark scheme all seen in responses.

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FURTHER STATISTICS A – AS UNIT 2

Overview of the Unit

Once again, this unit proved accessible to most candidates. In general, candidates performed as expected, with the exception of question 6, which proved to be very challenging for the majority of candidates. Only a small proportion of candidates were able to form the correct probability distribution required in question 6(c). Question 3 on the exponential distribution also proved challenging. However, the non-contextualised questions on random variables, and the question assessing Spearman's rank correlation coefficient were relatively well-answered.

Comments on individual questions/sections

Q.1 In general, this question was very well answered, as one might expect. It was the most successfully answered question on the paper and proved a positive start to the paper for many candidates. Some common errors included subtracting $5^2\mathrm{Var}(X)$, instead of adding this term; this led to an answer of -1344, which should have led candidates to question the validity of their answer.

Q.2 This was a twist on the classic 'find the equation of the regression line' question, which many candidates were not prepared for. Candidates seemed unable to apply the requisite pure mathematics skills from GCE Mathematics Unit 1. Many candidates tried to calculate S_{xy} and S_{xx}, which were not required.

Q.3 Once again this year, this question on the exponential distribution was poorly answered. It was the most poorly answered question on the paper. Candidates seemed to have trouble recognising the exponential distribution as well as using it. Many candidates incorrectly identified $\lambda = 2$, but did earn some credit for following through correctly in parts (a) and (b). The link between the Poisson distribution and the exponential distribution was poorly understood.

Q.4 This question was one of the better answered questions on the paper. Candidates were generally able to use the cumulative distribution function as required. Some common errors included substituting 0.75 into $F(x)$ in part (d), and integrating the cumulative distribution function.

Q.5 This was also a well answered question. The majority of candidates were able to score well in carrying out the hypothesis test in part (b)(i) and calculating Spearman's rank correlation coefficient in part (b)(ii). Interpreting the results in context proved a more challenging task.

Q.6 This was the second most poorly answered question on the paper. Parts (a) and (b) were relatively well answered. The straightforward nature of the hypothesis test meant that many candidates were able to earn marks for calculating the expected frequencies and the χ^2 test statistic. An obvious, common error was not combining classes for expected frequencies less than 5.

Part (c) proved challenging even for the most able candidates. Only a minority of candidates were able to successfully complete the various steps involved in calculating the expected daily net income. Some candidates recognised the distribution $B(22,0.9)$, but did not take into account the differing probabilities for the different number of groups that turned up.

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FURTHER MECHANICS A – AS UNIT 3

Overview of the Unit

The attempt rates for all questions indicate that there was sufficient time to complete the paper and it allowed candidates of all abilities to display their knowledge and demonstrate their skills. Many high scoring scripts with exemplar responses were seen. However, this paper appeared to be more challenging than the Summer 2022 paper. Questions 3 and 6 were the most demanding questions on the paper, with facility factors of less than 40.

Comments on individual questions/sections

Q.1 This was the most successful question on the paper and part (a) provided a gentle start to the paper. Therefore, as expected, it was answered successfully by the majority of candidates. In part (b), it was encouraging that almost all candidates correctly identified that the Conservation of Energy was required with three energy forms. However, many candidates did not consider the necessary two instances of elastic energy in their resulting equation.

Q.2 Almost all candidates scored full marks on part (a). In part (b), the most frequent errors were due to incorrect differentiation of $\sin\left(\frac{t}{2}\right)$ and $\cos\left(\frac{t}{2}\right)$. Unfortunately, many candidates arrived at expressions for the velocity of B with trigonometric functions that were not functions of t. For example,

$$\mathbf{v}_B = \frac{3}{2}\cos\left(\frac{1}{2}\right)\mathbf{i} + \frac{3}{2}\sin\left(\frac{1}{2}\right)\mathbf{j}$$

Q.3 A standard question on motion in a horizontal circle. There are no areas to highlight.

Q.4 The response to part (a)(i) was very disappointing, as a large number of candidates did not seem to understand what was initially required. Many candidates chose to begin with calculations for kinetic and potential energy, with some leaving out the term $1440\ \text{kJ}$ completely. A small number of candidates did not identify the k prefix as meaning kilo (Joules). Part (b) saw many fully correct solutions and so there is no area to highlight.

Q.5 Most candidates successfully calculated the required impulse in part (a), but many did not fully answer the question, as a direction was not provided.

In part (b), the use of signposting was much improved this year. Reassuringly, statements such as 'Using Conservation of Momentum' and 'Using Kinetic Energy' were frequently seen. Moreover, candidates seem to be comfortable in dealing with differences in kinetic energy in this style of question.

The most prominent issues were due to sign errors in setting up the kinetic energy equation and/or in the direction of motion of sphere B. Additional sign errors were also seen during algebraic manipulation.

Q.6 Efforts in this vertical circle question were disappointing. Candidates may have found the underlying 'real world' application of the question more challenging than was expected. The main difficulty was dealing with the potential energy in relation to the circular arc AB. Frequent sign errors were seen, resulting in many candidates unable to convincingly arrive at the result provided. The result for v^2 was provided in the question to allow accessibility to (ii), irrespective of earlier efforts. However, many candidates who were unsuccessful in (i), also struggled with the initial setup for Newton's second law in (ii), and hence did not attempt to substitute the v^2 that was provided.

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FURTHER PURE MATHEMATICS B – A2 UNIT 4

Overview of the Unit

The candidates performed very well on a high number of occasions and there were some excellent scripts. However, some candidates encountered difficulties with the requirements of some questions and poor algebraic skills were often seen, leading to low marks being awarded.

Comments on individual questions/sections

Q.1 This question was more poorly answered than expected. In part (a), some candidates seemed unfamiliar with composite functions, which was disappointing, as this topic is in the GCE Mathematics Unit 3 subject content. Some candidates gave the correct domain for fg, but were not precise enough with the range: $(1, \infty)$ was often seen rather than $[1, \infty)$. In part (b), most candidates were able to find the value of x to 3 decimal places; however, some candidates failed to take note of the domain and wrote ±1.662 as their final answer, losing the final accuracy mark. Some candidates decided to work with the exponential or logarithmic forms of $\cosh x$, which could lead to the correct answer, but candidates' responses often contained arithmetic errors.

Q.2 Both parts of this question were well answered, with the only errors in part (a) being arithmetic when calculating the cofactors. In part (b), there was an equal proportion of candidates either completing the square, calculating the discriminant, or solving the equation, but not all candidates were able to explain why the matrix was singular following this.

Q.3 Part (a) was well-answered, although some candidates should remember to show sufficient working in their responses, particularly when the answer is given in the question. In part (b), most candidates were able to expand $\left(z + \frac{1}{z}\right)^6$, but not all appreciated the need to halve their answer. Some candidates began by expanding $(\cos\theta + \mathrm{i}\sin\theta)^6$, and this approach could have led to the required form with extensive working, but few made progress.

Q.4 This question was well answered, but arithmetic errors meant some candidates lost accuracy marks unnecessarily. More than half the candidates opted to begin with row reductions, with some continuing to echelon form, whilst others formed simultaneous equations in two variables, before solving the equations. A significant proportion of candidates used the inverse matrix approach. A high level of success was seen with each of the methods.

Q.5 In part (a), the majority of candidates used the Maclaurin expansion of $\sin x$, replacing x with $2x$, but many candidates also chose to use the expansion of $f(x)$ – both methods proved successful for candidates. Part (b) was poorly answered. Many candidates tried to use the double-angle formulae for $\sin 2x$ to obtain an expression for $\cos x$, to be subsequently squared.

Few candidates seemed to be familiar with the idea of differentiating Maclaurin expansions to obtain other expansions; those who did, often earned full marks.

Q.6 All parts of this question were well answered. However, errors occurred in part (b) when differentiating using the product rule and simplifying $\sin\theta$ and $\cos\theta$ terms to give $\tan\theta$, which was disappointing, as these are GCE Mathematics Unit 3 skills. In part (c), some candidates made errors in manipulating the fractions to form an equation, whilst others divided by t, losing a set of coordinates.

Q.7 This question was well answered, with candidates choosing to write the complex number in either exponential form or trigonometric form. Disappointingly, errors occurred when calculating the modulus and argument of z, the required pre-requisite skills from Further Mathematics Unit 1.

Q.8 Most candidates answered this question very well. However, not all candidates were able to complete the square correctly. Other errors, although infrequent, were omitting the ½ in part (a), and π in part (b), or not giving their answers to the required level of accuracy.

Q.9 Although many candidates achieved full marks on this question, some candidates made careless algebraic errors, e.g. when dividing by $(x+1)^5$, candidates forgot to divide the constant of integration by this term. When little credit was given, this was often due to an incorrect integrating factor.

Q.10 This question proved to be a very challenging question for many candidates, with few fully correct solutions seen. In part (a), some candidates used the chain rule and the result in the Formula Booklet, rather than a proof of the derivative. There was an equal proportion of candidates using the different methods given in the mark scheme for part (a), but, in both instances, the final mark was often withheld, as very few candidates were able to justify why the positive square root was chosen. In part (b), candidates were not often successful in finding the required range of values. The most successful candidates realised that $2x+5$ must strictly lie between -1 and 1, whilst some candidates found the values of -2 and -3, but did not note the range correctly.

Q.11 In part (a), there was an equal proportion of candidates using the three methods given in the mark scheme, with each method proving successful for candidates. In part (b), most candidates spotted the need for partial fractions, but not all were able to find the correct values in their expression. Of those that did, not all could split the $\frac{ax+b}{x^2+9}$ term into two terms in order to proceed with the required integration. However, it was pleasing to see many fully correct solutions for this question.

Q.12 This question was well answered. Common errors were dividing throughout by $\cos\theta$ and thereby losing some solutions, or multiplying the general solutions of 3θ by 3, instead of dividing by 3.

Q.13 This question was answered better than expected, with full credit given on many occasions. In part (a) there was a variety of methods used to reach the required result. The general solution of x often followed correctly, although some candidates noted this as y in terms of x rather than x in terms of t, and consequently lost a mark. In part (b), most candidates realised that they needed to use the general solution of x and the differential equations given in the stem of the question, but some repeated the processes used in part (a)(i), only to arrive at a similar result as that in (a)(i). In part (c), errors were often in the arithmetic. Candidates who made errors in parts (a) and (b) often found that they had more complex expressions to deal with in part (c).

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FURTHER STATISTICS B – A2 UNIT 5

Overview of the Unit

It was encouraging to see that the standard of mathematics, demonstrated by candidates, on this paper was the most pleasing out of all the GCE Mathematics and GCE Further Mathematics statistics papers. Candidates' performance on the 2023 paper was comparable to previous series and was a little more consistent than last summer (2022). There were a good number of candidates scoring well over 70 marks. There appear to be more better performing candidates in 2023, than in previous series. In general, candidates were adept at answering the questions on parametric tests. Calculating the confidence interval in question 3 was well done, but calculating the confidence level in question 7 was less well done. Question 5 on combinations of independent random variables was generally well done.

Comments on individual questions/sections

Q.1 This question should have been a straightforward start to the paper, and, for many candidates, it was. The error that many candidates made was not using the t-distribution in part (b). Despite having calculated an unbiased estimate for the variance in part (a), these candidates continued to use a normal distribution. The vast majority of candidates were able to give sensible answers in part (c).

Q.2 This was a lengthy question, which some candidates left until the end to answer. Part (a) was generally well done. In part (b), many candidates divided through by a without referencing $a > 0$, so did not earn all the marks available. The most common errors in part (c) were not using 10^2 as a denominator, using σ^2 for $\mathrm{Var}(\bar{X})$ and $\mathrm{Var}(\bar{Y})$, instead of $\frac{\sigma^2}{20}$ and $\frac{k\sigma^2}{25}$. As expected, $k = \frac{5}{6}$ often appeared in the absence of sufficient and convincing workings. Part (e) caused the most difficulty, with few candidates being able to find a correct expression for $\mathrm{Var}(T_3)$. Of those that did, many candidates did not differentiate their expression before setting it equal to 0. Only a handful of candidates justified that T_3 was a minimum.

Q.3 The confidence interval in part (a) was calculated correctly by the majority of candidates. There were a few incorrect z-values used and some candidates calculated the standard error incorrectly, but, on the whole, this question was answered well. Part (c) was done reasonably well, with a few omissions of $\sqrt{n}$ and incorrect values used for $\bar{x}$.

Q.4 Although this question was the most well answered on the paper, the quality and preciseness of the hypotheses was often lacking. The most succinct way to write the hypotheses was as $H_0: \eta_d = 0$, $H_1: \eta_d > 0$, where η_d is the median difference in numbers of followers before and after appearing on the show.

Q.5 Most candidates were able to answer part (a) correctly. Part (b) was also relatively successful with only a small number of candidates unable to standardise or select a correct z-value, with $z = -2.4572$ often being chosen. Unfortunately, there were several candidates who were unable to deal with $T = X_1 + X_2 + X_3 + Y_1 + Y_2 + Y_3 + Y_4$ correctly to find the variance, with $\mathrm{Var}(T) = 3^2 \times 100 + 4^2 \times 36 = 1476$ being the common error.

Q.6 Once again, this question was well answered, but, as in question 4, the quality and preciseness of the hypotheses was often lacking. The most succinct way to write the hypotheses was as $H_0: \eta_1 = \eta_2$, $H_1: \eta_1 > \eta_2$, where η_1 and η_2 are the median number of races entered by club members and non-members respectively. Another common error was to ignore the non-member who raced 0 times. This clearly arose from a misconception, where ignoring the difference of 0 in question 4 was the correct thing to do.

Q.7 This was the most poorly answered question on the paper. Many candidates, though not all, were able to correctly calculate the standard error for the difference of means. Only a few candidates were able to form an equation in k in order to find the z-value. Fewer still were able to progress further from $k = 2.333$ .... Those that did manage to find the confidence level, usually did so using their calculator. Most candidates were able to give a sensible assumption in part (b), although “riding both bikes at the same time” was not awarded a mark.

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FURTHER MECHANICS B – A2 UNIT 6

Overview of the Unit

The paper was well received by most candidates, but it appeared to be less accessible than the Summer 2022 paper. As usual, many high scoring scripts with exemplar responses were seen. There was no evidence to suggest that candidates found the paper too long to complete in the allocated time, as most candidates managed to attempt all the questions on the paper. Questions 5 and 6 were the most demanding questions on the paper, with facility factors of less than 50.

Comments on individual questions/sections

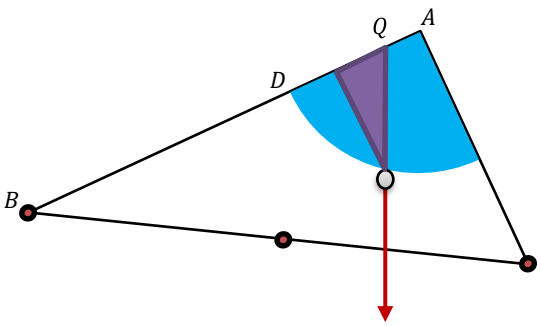
Q.1 Candidates continue to be adept at answering questions assessing the topic of rigid bodies and hence many fully correct solutions to parts (a) and (b) were seen. Only a handful of candidates provided exact forms for answers to parts (a) and (b), although this was not a requirement of the question. Many candidates were less successful in securing the final mark in part (c), as they were often unable to provide an articulate reason to support their answer.

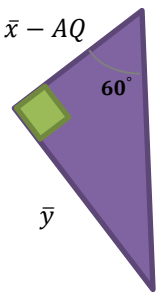
Q.2 The response to this question was not as good as expected. Unfortunately, some candidates did not know the formula for the volume of a cone. Furthermore, a small number of candidates were unaware that the formula for the volume of a cone could simply be stated, unless instructed otherwise. Consequently, integration was often used to find the volume and, due to the complexity of the method, almost all of these attempts were unsuccessful and cost candidates valuable examination time.

Q.3 Candidates have always demonstrated a strong understanding of Simple Harmonic Motion and this question, which was set in context, was no exception.

Q.4 This was the most successful question on the paper. It was reassuring to see that candidates were not troubled by the algebraic context of the question.

Part (c) provided the greatest challenge since, despite most candidates considering a simple sketch, many were still unable to identify the appropriate triangle, as shown below.





Q.6 This was the first time that a second-order differential equation has appeared on Unit 6. Efforts in parts (a) and (b) were generally very good. Part (c) was less successful, with most errors occurring due to incorrect differentiation of candidates' expression for x. In part (d), many candidates recognised that the damping was 'critical', but did not provide a legitimate reason to support their answer.

Supporting you

Useful contacts and links

Our friendly subject team are on hand to support you between 8.30am and 5.30pm, Monday to Friday.
Tel: 029 2240 4251
Email: mathematics@wjec.co.uk
Qualification webpage: https://www.wjec.co.uk/qualifications/mathematics-a-as-level/

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[i] ***Please note that where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.***