GCSE (9-1)

WJEC Eduqas GCSE (9-1) in

GEOLOGY

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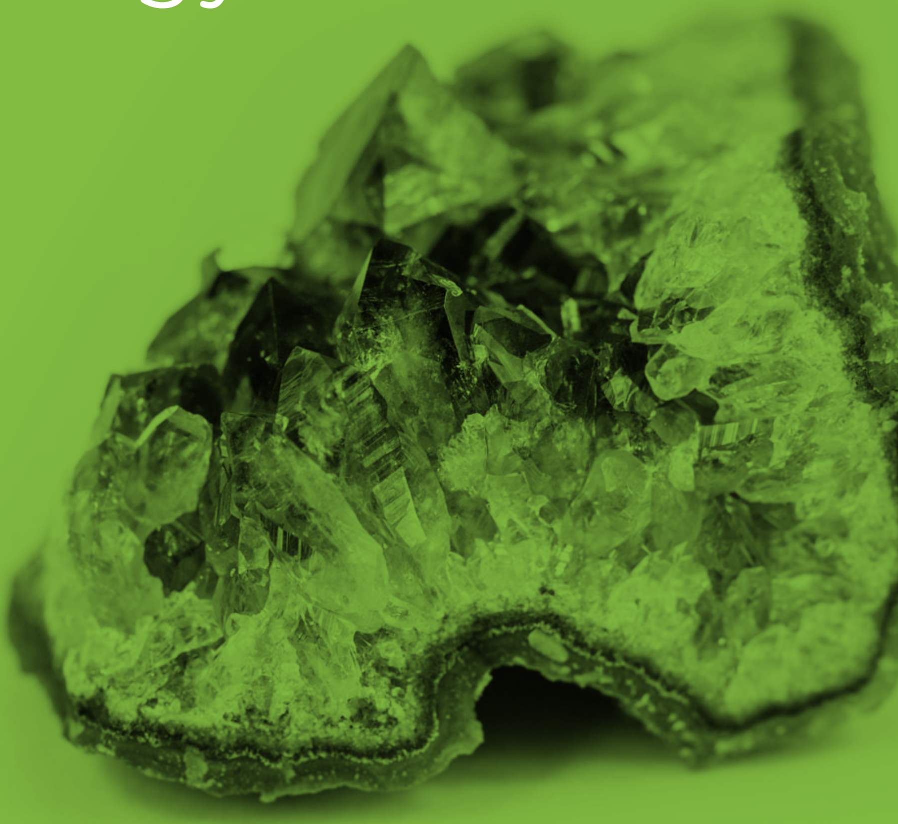
Mathematical Guidance for GCSE Geology

Teaching from 2017
For award from 2019



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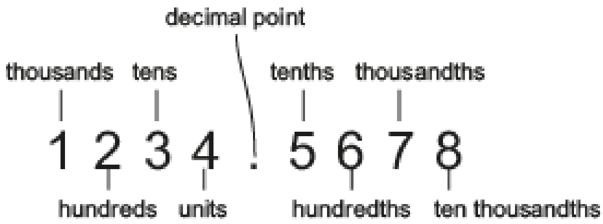
Introduction

Mathematical skills are essential in all of the GCSE science qualifications including geology. The following pages document some of the more important mathematical skills that are required of students following the WJEC Eduqas Geology GCSE course. Further details on these and other topics can be found at http://www.ase.org.uk/resources/maths-in-science/.

Students will require the use of a scientific calculator in their lessons and in the examinations.

Decimal and standard form

All the numbers we use to describe quantity, size etc. in geology can be written in **decimal form**, with an arbitrary number of decimal places. Decimal integers written to the right of the decimal point specify the number of tenths, hundredths, thousandths and so on. In the same way as the integers to the left of the decimal point indicate the number of units, tens, hundreds and so on.



$$1234.5678 = 1000 + 200 + 30 + 4 + \frac{5}{10} + \frac{6}{100} + \frac{7}{1000} + \frac{8}{10000}$$

In geology we often encounter both very large and very small numbers e.g. the age of the Earth and the diameter of a clay mineral. It is impractical to write the age of the Earth as 454 000 000 000 years old or the diameter of a clay mineral as 0.0000039 m. so **standard index form** is used instead. This is the conventional way of writing both large and small numbers and has the additional advantage of simplifying calculations by enabling the use of the index laws. In standard index form the decimal form is expressed as a number between 1 and 10 multiplied by 10 to the appropriate index (power).

To convert large numbers to standard index form, count the zeros in the number. This gives the value of the index of 10. Then make any adjustment required so that the number in front lies between 1 and 10.

e.g. 4 540 000 000 years is 454 followed by 7 zeros = $454 \times 10^7 = 4.54 \times 10^9$ years

To convert small numbers to standard index form, count the zeros in the number, including the zero before the decimal point. This gives the value of the index of 10, which will be negative if the number is less than one. The digits in front of the index should be written to lie between 1 and 10.

e.g. 0.00000391 m is 6 zeros followed by 391 = 3.91×10^{-6} m

As an alternative to standard index form, in the **S.I. system** of units different names are introduced for each thousandfold increase or decrease in size. For example, the basic unit of length is the metre. The next unit up, one thousand times larger, is called the kilometre and the next unit down, one thousand times smaller, is called the millimetre. The prefixes used in S.I. units are listed in the table opposite.

multiple	**prefix**	**symbol**	**example of units**
10^{-9}	nano	n	nanometre
10^{-6}	micro	µ	micrometre
10^{-3}	milli	m	millimetre
1	no prefix		metre (m)
10^{3}	kilo	k	kilometre (km)
10^{6}	mega	M	megametre (Mm)
10^{9}	giga	G	gigametre (Gm)

A clay mineral with a diameter of 0.00000391 m could therefore written as 3.91×10^{-6} m or 3.91 µm.

Significant figures

Significant figures are 'each of the digits of a number that are used to express it to the required degree of precision, starting from the first non-zero digit'. Numbers are often rounded to avoid reporting insignificant figures. For example, it would create false precision to express a measurement as 12.34500 kg (which has seven significant figures) if the scales only measured to the nearest gram and gave a reading of 12.345 kg (which has five significant figures).

Non-zero figures are always significant. Thus, 22 has two significant figures, and 22.3 has three significant figures. With zeroes, the situation is more complicated:

a. Zeroes placed before other figures are not significant; 0.046 has two significant figures.
b. Zeroes placed between other figures are always significant; 4 009 has four significant figures.
c. Zeroes placed after other figures but behind a decimal point are significant; 7.90 has three significant figures.
d. Zeroes at the end of a number are significant only if they are behind a decimal point as in (c). Otherwise, it is impossible to tell if they are significant. For example, in the number 8 200, it is not clear if the zeroes are significant or not. The number of significant figures in 8 200 is at least two, but could be three or four. To avoid uncertainty, use standard index form to place significant zeroes behind a decimal point:

8.200×10^3 has four significant figures; 8.20×10^3 has three significant figures; 8.2×10^3 has two significant figures.

In a calculation involving multiplication, division, trigonometric functions, etc., when asked to round to an appropriate level of accuracy, the number of significant figures in an answer should equal the least number of significant figures in any one of the numbers being multiplied, divided etc.

For example, the mass of a granite pebble is determined as 276.5 g (four significant figures) and its volume is 105 cm^3 (three significant figures). The density of the pebble is

$\frac{276.5}{105} = 2.63$ g/cm^3 (three significant figures).

When quantities are being added or subtracted, the number of decimal places (not significant figures) in the answer should be the same as the least number of decimal places in any of the numbers being added or subtracted.

When doing multi-step calculations it is good practice to keep at least one more significant figures in intermediate results than needed in your final answer. For instance, if a final answer requires two significant figures, then carry at least three significant figures in calculations. If you round-off all your intermediate answers to only two significant figures, you are discarding the information contained in the third significant figure, and as a result the second significant figure in your final answer might be incorrect. (This phenomenon is known as a "rounding error.")

Order of magnitude calculations

Simply speaking an order of magnitude is how many powers of ten there are in a number. Orders of magnitude can be determined easily when a number is written in standard form. For example, 237 (2.37×10^2) and 823 (8.23×10^2) both have an order of magnitude of 2.

Comparing orders of magnitude is a useful way of estimating the difference between two numbers. For example, a scanning electron microscope may produce a magnification of up 40 000 (4×10^4) whereas a typical optical laboratory microscope may produce a magnification of just 40 (4×10^1). A scanning electron microscope therefore produces an image 4 orders of magnitude bigger than the object and 3 orders of magnitude bigger than an optical microscope.

Ratios, fractions and percentage

Ratios, fractions and percentages are some of the most useful math concepts in geology as they enable comparisons to be made between the sizes of many different geological phenomena.

Mineral/ hardness		*Common equivalent*
Diamond	10	
Corundum	9	
Topaz	8	
Quartz	7	
Orthoclase feldspar	6	← steel pin
Apatite	5	
Fluorite	4	← copper coin
Calcite	3	← finger nail
Gypsum	2	
Talc	1	

The above table of Mohs hardness scale can be used to show the link between ratios, fractions and percentages.

For example, three of the minerals out of the ten can be scratched by a copper coin which as a fraction is $^{3}/_{10}$ which could be re-written as $^{30}/_{100}$ or 30%. The proportion (a part to whole comparison) of minerals that can be scratched by a copper coin can be expressed as $^{3}/_{10}$ or 30% or 3 in 10.

The ratio (a part to part comparison) of minerals that can be scratched by a copper coin to those that cannot is 3:7. In comparison five of the minerals out of the ten can be scratched by a steel pin which as a fraction is $^{5}/_{10}$ (which simplifies to ½) or 50%. The ratio of minerals that can be scratched by a steel pin to those that cannot is 5:5 (which simplifies as 1:1).

Manipulating and solving simple algebraic equations

An equation is a statement that the values of two mathematical expressions are equal. The most simple types of equations are those that when plotted on a Cartesian grid (x and y axis) produce a straight line. None of the variables in these equations are squared, cubed etc. and the variables are represented by symbols.

Scientists commonly need to know how quickly or slowly ("at what rate") a given process is occurring. A rate is a measurement of change relative to time. Many geological processes approximate to simple linear relationships (i.e. constant rates of change) and therefore would plot as straight line graphs on a Cartesian grid.

A rate (r) is calculated by determining the amount of change (for example, distance travelled) and the time elapsed. To do this, we need two values for time (t_1 and t_2) and two corresponding values for the condition that is changing (d_1 and d_2). So for example:

$$r = \frac{d_2 - d_1}{t_2 - t_1}$$

where d_2 is the distance travelled at time t_2 and d_1 is the distance travelled at time t_1. The Greek letter Δ, "delta," means change, and you may often see it used in rate calculation problems. Written using delta, our example rate equation becomes:

$$r = \frac{\Delta d}{\Delta t}$$

This simple linear algebraic equation can be applied to calculate the rate (speed) of plate motion. In 2006, geologists working with the Plate Boundary Observatory Network began closely tracking the location of a GPS station west of the San Andreas Fault in California. The station, which is located on the Pacific Plate, is moving slowly northwest past the North America Plate. In May 2007, researchers recorded the station 33 mm northwest of its original position. In May 2012, they recorded it 195 mm northwest of its original position. To calculate the rate of motion of the station (and thus the Pacific Plate) between 2007 and 2012 firstly the total displacement (Δx) needs to be determined,

$$\Delta x = x_2 - x_1$$

$$\Delta x = 195 - 33 = 162 \text{ mm}$$

Since the time period of interest is between 2007 and 2012, we know that Δt = 5.00 years. Therefore:

$$r = \frac{\Delta x}{\Delta t}$$

$$r = \frac{162}{5.00} = 32.4 \text{ mm/year}$$

So between 2007 and 2012 the Pacific Plate has a rate of motion (speed) of 32.4 mm/year or a velocity of 32.4 mm/year to the northwest.

If the plate continues to move at the same rate in the same direction, then it is possible to calculate how far it will be from its original (May 2006) position. Therefore, by May of 2050, rearranging the equation:

$$r = \frac{\Delta x}{\Delta t}$$

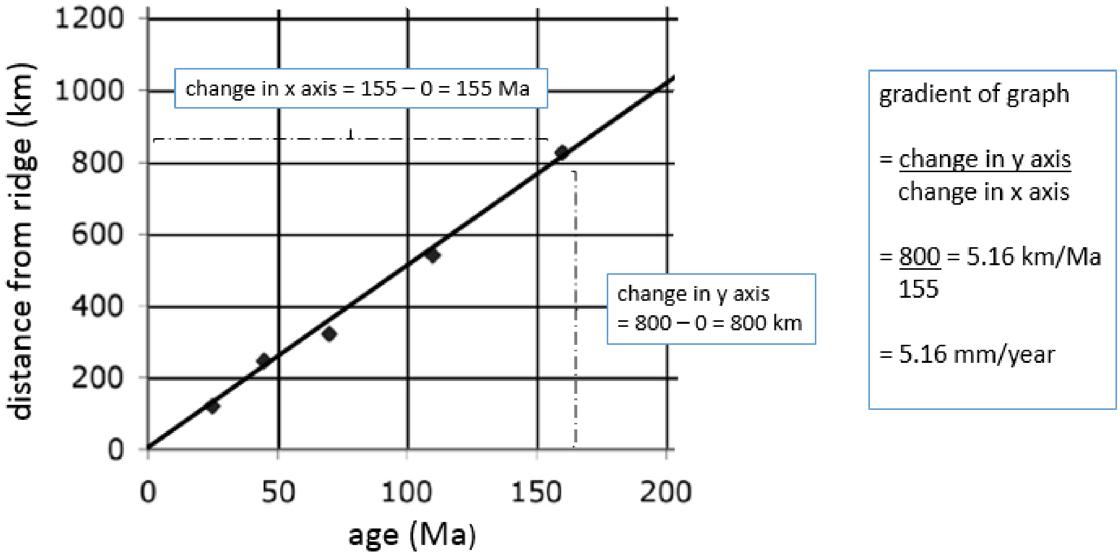
to make Δx the subject of the equation so that:

$$\Delta x = r \times \Delta t$$

then, Δx = 32.4 × (2050 – 2006) = 32.4 × 44.00 = 1 430 mm

Therefore by May 2050, the station will have moved 1.43 m if the speed and direction remained constant.

Sometimes a series of readings of distance and time may be taken and plotted on a Cartesian grid as below. In this case the best-fit linear plot passes through the origin, hence time and distance are proportional and show a strong positive correlation.

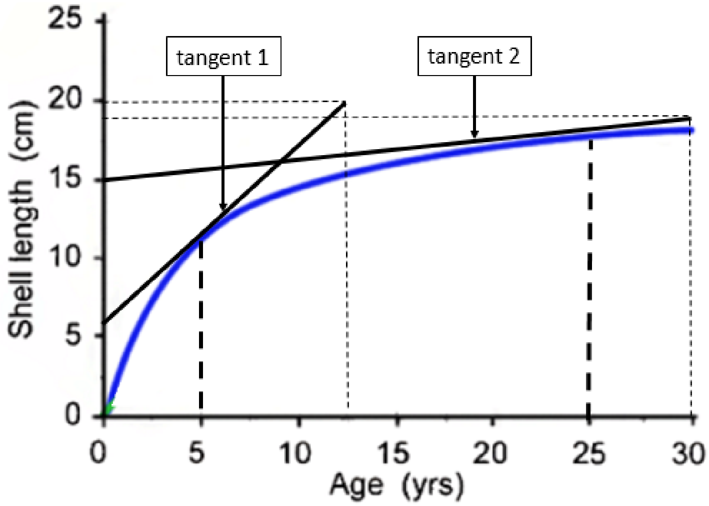


The rate of change (speed) of the oceanic crust can, as before, be calculated using:

$$r = \frac{\Delta x}{\Delta t}$$

but in this case as distance is on the y-axis and time on the x-axis then the rate of change is equivalent to the gradient of the best-fit straight line. Consequently the mean rate of spreading of the oceanic crust on one side of the ridge in this example is 5.16 mm/year.

However, not all geological processes occur at constant rates. When these relationships are plotted on a Cartesian grid the resulting plot is a curve. The graph below shows how the length of the shell of a mollusc (bivalve) increases with time. There is a positive relationship between the two variables (i.e. as time increases so does shell length) but there is no correlation between the two variables. Despite the fact that the line passes through the origin the two variables are not proportional as the line is curved and not straight.



<u>tangent 1</u>
rate of shell length growth at t = 5 years is

$= \frac{20 - 6}{12.5 - 0} = 1.1$ cm/year

<u>tangent 2</u>
rate of shell length growth at t = 25 years is

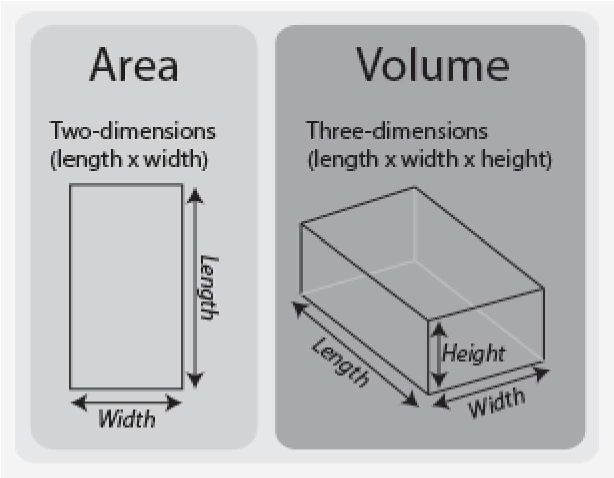
$= \frac{19 - 15}{30 - 0} = 0.13$ cm/year

In such circumstances the instantaneous rate of change (the rate of shell length growth) can be determined by drawing a tangent to the curve at the time of interest. A tangent to a curve is a line drawn so that it just touches the curve without crossing it. Two such tangents are drawn at times = 5 and 25 years on the graph above. The instantaneous rate of shell length growth at these two times are 1.1 and 0.13 cm/year and are calculated by determining the gradient of the tangent to the curve (assuming time is the independent variable). Note that the faster the rate of change the steeper the gradient of the tangent to the curve.

Calculating the area and volume of regular shapes

The ability to calculate the area of shapes such as triangles and rectangles as well as the volume and surface area of cuboids is used in many areas of geology.

Area, volume and surface area of rectangles and cuboids



Area formula; Skills You Need goo.gl/wmvkv3

Volume formula; Skills You Need goo.gl/wmvkv3

The area of the rectangle = length × width

The volume of the cuboid = length × width × height

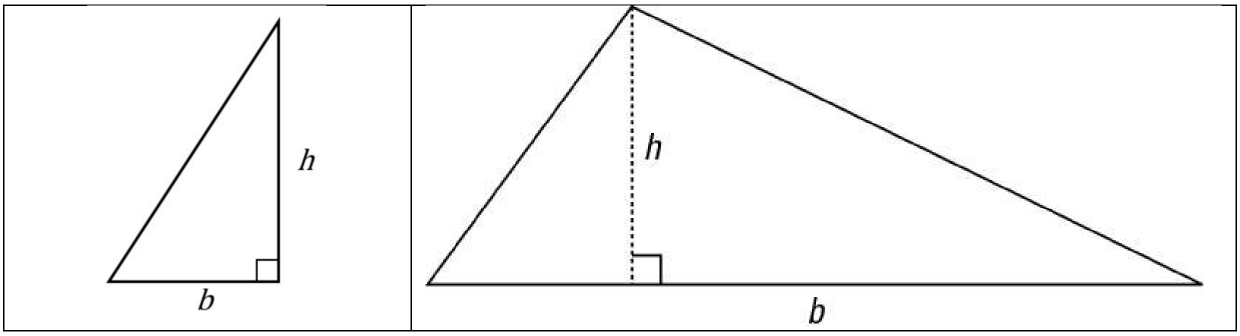
The surface area of the cuboid = 2 × ((length × width) + (width × height) + (length × height))

Area of triangles

The method used to calculate the area of a triangle depends upon the type of triangle and the information available. For triangles where the base and perpendicular height are known:

The area of the triangle = ½ × base × perpendicular height

For right angle triangles the calculation is relatively simple as the perpendicular height is one of the sides of the triangle but in non-right angle triangles the perpendicular height may have to be calculated.



Calculating squares and square roots

A number multiplied by itself is called a *square* and is written in the form x^2.

e.g $4^2 = 4 \times 4 = 16$

$6^2 = 6 \times 6 = 36$

If a square shape has sides of length 7 m, the area = 7×7, or $7^2 = 49\,\text{m}^2$

The *square root* of a number must be squared (that is multiplied by itself or raised to the power 2) to give the number.

e.g The square root of 16 = 4 (because 4 must be multiplied by itself to give 16).

It is written $\sqrt{16} = 4$

The square root of 64 = 8 (because 8 must be multiplied by itself to give 64).

It is written $\sqrt{64} = 8$

If a square shape has an area of $25\,\text{m}^2$, the length of each side is $\sqrt{25} = 5\,\text{m}$

Data and Statistical Analysis

Designing a good experiment for a fieldwork investigation so that the information yields meaningful statistics is extremely important. Integral to this is a good sampling method.

Sampling methods

Sampling should be conducted in a way that will best represent the data being collected. There are three main types of sampling: random, systematic and stratified.

In **random sampling** every item has an equal chance of being selected. For many studies this is the most desirable approach as there is no bias. The most common way of random sampling is to use a random number table or generator.

In **systematic sampling** there is some structure or underlying order to the way in which the data is selected. For example, this may be achieved by a gridding (quadrat) system or by line transects.

With **stratified sampling** the population is purposely split into separate groups/layers (strata). Then each group is further analysed by random or systematic sampling. This approach could be adopted to investigate the particle shapes in a sieved unconsolidated sediment for each grain size fraction.

Data Analysis

Univariate analysis is the simplest form of analysing data as it deals with just one variable. Looking at two variables at one time is termed bivariate analysis.

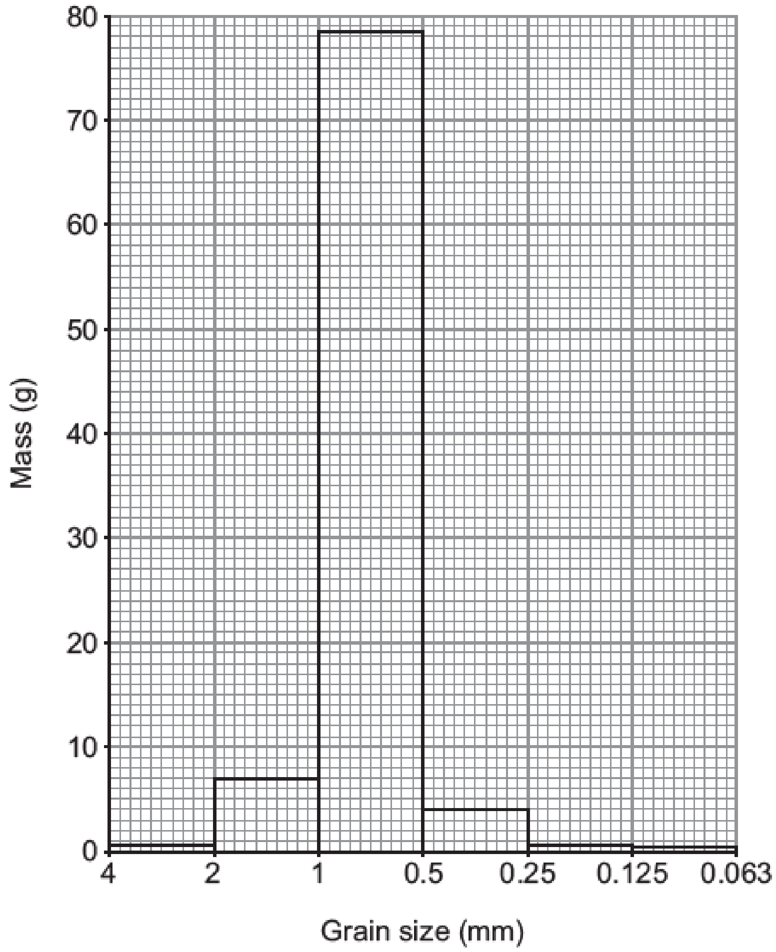
Univariate analysis

The first step in the analysis of univariate data is the production of a frequency table. Depending on the data type there are several choices for displaying the variable being investigated. Candidates are expected to be able to construct and interpret both bar charts and rose diagrams.

The frequency table below provides information for a sieved modern beach sand.

grain size (mm)	mass (g)
4≤mm<8	0.0
2≤mm<4	0.3
1≤mm<2	7.0
0.5≤mm<1	78.5
0.25≤mm<0.5	4.0
0.125≤mm<0.25	0.2
0.063≤mm<0.125	0.1
0.032≤mm<0.063	0.0

The most appropriate way of displaying this data is as a bar chart. The bar chart shows that the modal class is 0.5≤mm<1 and that the sediment is well sorted.



Orientated data is an important category of geological information. Perhaps the most important orientated data in geology is strike and dip direction. Such data can be successfully represented on a type of circular plot called a rose diagram. In a rose diagram the azimuth (a horizontal bearing between 000-360^o measured clockwise from North) is plotted around the circumference of the circle and the frequency is plotted radially from the centre of the circle. Rose diagrams avoid the disadvantage of conventional bar charts in that values that lie close together (e.g. 359^o and 001^o) are plotted close together.

The data below are dip direction measurements from westerly dipping bedding planes in folded Carboniferous limestones exposed at Ecton in Staffordshire.

242	252	260	251	272	268	248	259	269	258
254	244	242	254	271	267	250	257	268	259
250	248	240	255	248	270	272	265	251	247
246	257	230	258	249	272	276	267	252	262
254	260	234	243	244	262	272	266	262	255

Ranking these values enables the median (the value that evenly splits the number of observations into a lower half of smaller observations and an upper half of larger measurements) of the data set to be determined. As there is an even number of results the median is the mean of the two central numbers i.e ½(255 + 257) = 256^o.

230	243	248	250	254	***257***	259	262	268	272
234	244	248	251	254	257	260	265	268	272
240	244	248	251	254	258	260	266	269	272
242	246	249	252	255	258	262	267	270	272
242	247	250	252	***255***	259	262	267	271	276

To find the mean, the values in the data set are added and then divided by the number of values that were added, i.e.

$$\bar{x} = \frac{x_1 + x_2 + ... + x_n}{n}$$

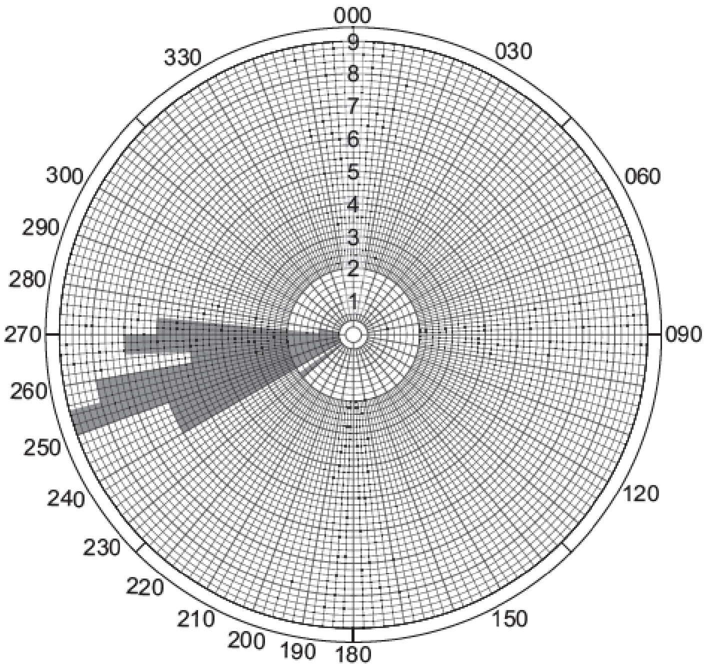
$$= \frac{230 + 234 + ... + 276}{50} = \frac{12812}{50} = 256^\circ$$

The mode (the most frequently occurring value) of the raw data is 272^o.

The next useful step in the interpretation of this data is to produce a frequency table by dividing the data into class intervals, customarily of the same width, to provide a count of the relative frequencies of the classes. This will then enable a visualisation of the data in a rose diagram.

Dip direction (DD) class		Frequency
1	230^o ≤DD<235^o	2
2	235^o ≤DD<240^o	0
3	240^o ≤DD<245^o	6
4	245^o ≤DD<250^o	6
5	250^o ≤DD<255^o	9
6	255^o ≤DD<260^o	8
7	260^o ≤DD<265^o	5
8	265^o ≤DD<270^o	7
9	270^o ≤DD<275^o	6
10	275^o ≤DD<280^o	1

The modal class for the data is therefore 250^o ≤DD< 255^o which is especially clear in the rose diagram below.



000
9
8
7
6
5
4
3
2
1
030
060
090
120
150
180
190
200
210
220
230
240
250
260
270
280
290
300
330

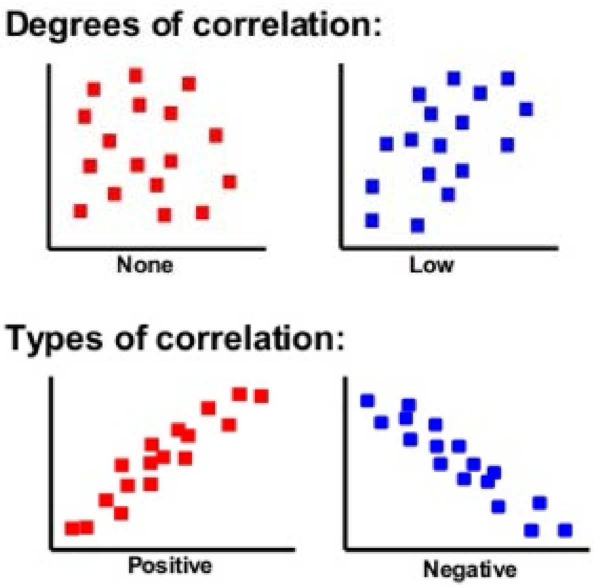
Bivariate analysis

A dataset that contains two variables is termed bivariate data. In geology there is often an interest in comparing two measurements made for the same site e.g. groundmass crystal size and distance from the edge of a pluton. Among the many commonly used graphical techniques used to analyse and display bivariate data, perhaps the most frequently utilised is the scatter diagram.

Scatter diagrams

Scatter diagrams are used to show graphically the relationship between two variables. Two axes are drawn in the usual way with the variable that is believed to cause the change in the other (the so-called independent variable) plotted on the x-axis; the dependent variable is therefore plotted on the y-axis.

By studying the resulting pattern of the pairs of data on the scatter diagram the degree of correlation may be evident. Correlation gives an idea of how strong the linear relationship between the bivariate data is (e.g. for the curved data no correlation exists). Commonly encountered patterns include:



Scatter Diagram - correlation; ABB Group goo.gl/PhgWx4

It is very common for graphs of the relationship between pairs of geological variables to be well approximated by straight lines. However, the fit is never perfect. Despite this fact it may be possible to draw in by eye, a trend or best fit straight line, which should appear to pass as close as possible to all the points plotted (with care taken to exclude obvious anomalies). The best fit straight line does not need to pass through the origin but it is good practice that the line of best fit should pass through the double mean point ($\bar{x},\bar{y}$) i.e. the point that is mean of x values:mean of y values.

As an example data on how the number of days in a year has varied during the past 900 million years of Earth history is presented on the next page.

Time (Ma)	Number of days in year
0	365
70	370
220	372
290	383
340	398
380	399
395	405
410	410
420	400
430	413
440	421
450	414
510	424
600	417
900	486
mean = 390	mean = 405

Earth Rotation Changes and the Length of the Day; www.nasa.gov

A scatter graph for the data demonstrates a positive high degree of correlation between the two variables, but as the best fit straight line does not pass through the origin, the two variables are not proportional.