



**For teaching from 2010  
For awards from 2011**

# **WJEC LEVEL 2 CERTIFICATE IN ADDITIONAL MATHEMATICS**

## **SPECIMEN ASSESSMENT MATERIALS**



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Candidate Name	Centre Number	Candidate Number



**WJEC LEVEL 2 CERTIFICATE IN  
ADDITIONAL MATHEMATICS  
SPECIMEN PAPER**

**SUMMER 2011**

$2\frac{1}{2}$  hours

**ADDITIONAL MATERIALS**

A calculator will be required for this paper.

**INSTRUCTIONS TO CANDIDATES**

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

Take  $\pi$  as  $3 \cdot 14$  or use the  $\pi$  button on your calculator.

**INFORMATION FOR CANDIDATES**

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded that assessment will take into account the quality of written communication (including mathematical communication) used in your answer to question **15**.

When you are asked to show your working you must include enough intermediate steps to show that a calculator has not been used.

For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1	7	
2	8	
3	5	
4	4	
5	7	
6	7	
7	5	
8	4	
9	7	
10	5	
11	6	
12	5	
13	7	
14	5	
15	5	
16	4	
17	9	
<b>TOTAL MARK</b>		

1. (a) Showing all your working, find the value of each of the following.

(i)  $25^{\frac{3}{2}} \times 27^{-\frac{4}{3}}$

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[2]

(ii)  $\left(16^{\frac{1}{4}}\right)^{-4}$

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[1]

(b) Showing all your working, simplify each of the following.

(i)  $\frac{5y^{\frac{3}{2}} \times 4y^{-\frac{3}{4}}}{(y^3)^{\frac{1}{4}}}$  ,

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(ii)  $\frac{2x^{\frac{1}{3}} + 5x^{\frac{4}{3}}}{4x^{\frac{1}{3}}}$  .

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[4]

2. (a) Find the remainder when  $3x^3 - x^2 + 5x + 42$  is divided by  $x + 2$ .

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(b) (i) Show that  $x - 4$  is a factor of  $2x^3 - 3x^2 - 23x + 12$ .

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(ii) **Hence** factorise  $2x^3 - 3x^2 - 23x + 12$ .

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3. Find  $\frac{dy}{dx}$  for **each** of the following.

(a)  $y = 7x^5 + x - 18$

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..... [3]

(b)  $y = x^{-6}$

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..... [1]

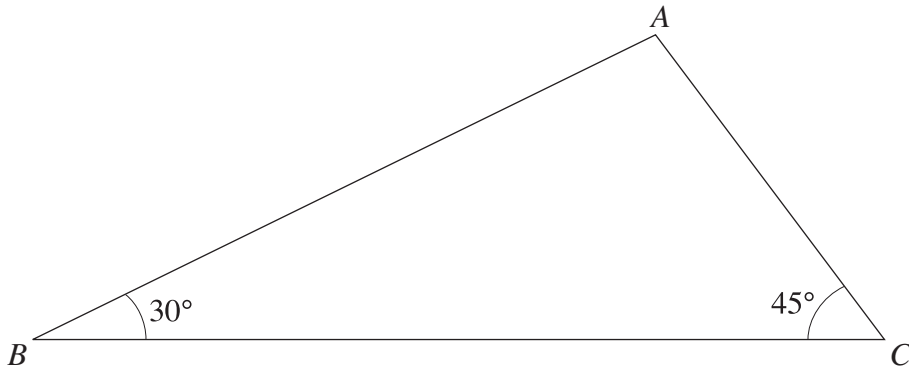
(c)  $y = x^{\frac{2}{3}}$

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..... [1]



4. The diagram shows triangle  $ABC$  with  $\hat{A}BC = 30^\circ$  and  $\hat{A}CB = 45^\circ$ .



*Diagram not drawn to scale.*

The perpendicular distance from  $A$  to  $BC$  is 5 cm. Show that  $BC = 5(\sqrt{3} + 1)$  cm .

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5. Find the coordinates of the stationary points on the curve  $y = x^3 - 3x - 2$  and determine their nature. You must show all your working.

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[7]

- 6. A square and a rectangle are such that the side of the square is equal in length to the **shorter** side of the rectangle. The sum of the areas of the square and the rectangle is  $198\text{ cm}^2$ , and the sum of the perimeters is  $80\text{ cm}$ . Calculate the dimensions of the rectangle.

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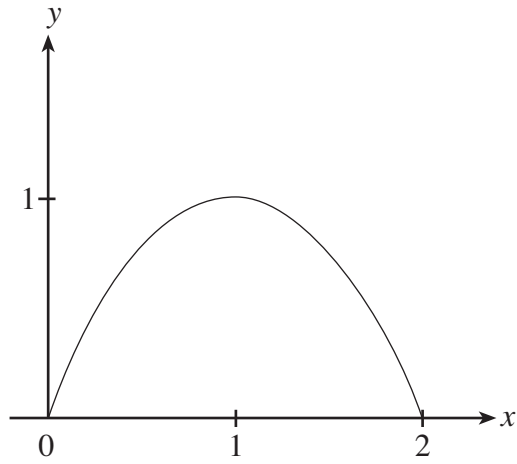
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[7]

7. The diagram shows the curve  $y = 2x - x^2$ .



Calculate the area of the region bounded by the curve  $y = 2x - x^2$  and the  $x$ -axis.

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8. Solve the equation  $\frac{2}{2x+3} + 4 = \frac{4x}{x-2}$  .

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9. (a) Factorise  $12x^2 + 11x - 15$ .

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**Hence** solve the equation  $12x^2 + 11x - 15 = 0$ .

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- (b) Use the method of completing the square to find the least value of  $x^2 + 16x + 3$ .

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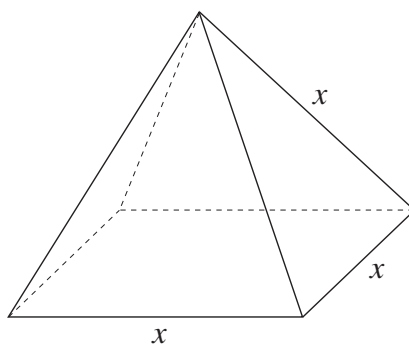
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[3]

10. The diagram shows a square based pyramid. The length of each of the edges is  $x$ .



*Diagram not drawn to scale.*

Show that the total surface area of the pyramid is  $x^2(1 + \sqrt{3})$ .

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11. Find the equation of the tangent to the curve  $y = x^3 - 3x^2 + 2$  at the point on the curve where  $x = -1$ .

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12. The coordinates of the points  $P$  and  $Q$  are  $(2, 3)$  and  $(14, 19)$  respectively.

(a) Calculate the length of the line  $PQ$ .

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(b) Find the gradient of a straight line perpendicular to  $PQ$ .

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13. (a) Given that  $y = x^2 - x$ , find  $\frac{dy}{dx}$  from first principles.

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- (b) Find the  $x$ -coordinate of the point on the curve  $y = x^2 - x$  where the gradient of the tangent to the curve is 3.

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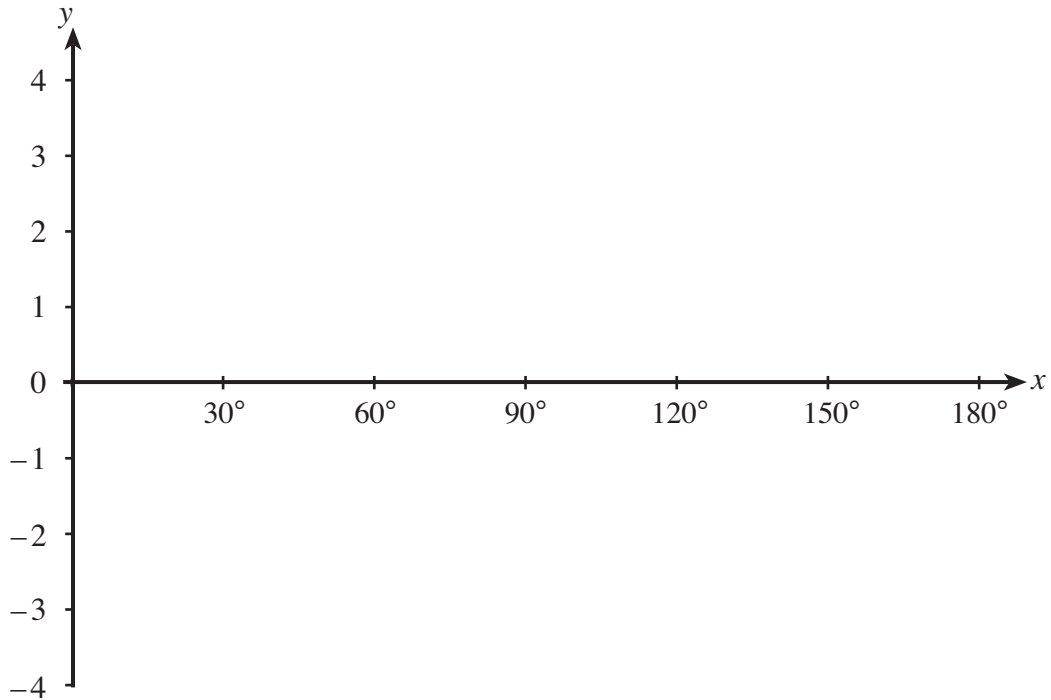
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[2]

14. (a) On the axes below, sketch the graph of  $y = 2 \sin 3x$  for values of  $x$  from  $0^\circ$  to  $180^\circ$ .



[2]

- (b) Find all the solutions of the equation  $2 \sin 3x = -1$  for values of  $x$  from  $0^\circ$  to  $180^\circ$ .

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[3]

15. You will be assessed on the quality of your written communication in this question.

Jodie takes part in a mathematics quiz. She is given the following clues to find an equation of a straight line.

The equation is of a straight line that:

- passes through the origin
- is not parallel to the  $x$ -axis
- is perpendicular to the straight line with equation  $4x + y - 3 = 0$
- intersects the  $y$ -axis only once

Find the equation for Jodie and clearly indicate which of the clues are essential in order to work out the answer.

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16. Prove that  $\frac{2x}{5} + \frac{x-1}{6} + \frac{3x+5}{10} \equiv \frac{13x+5}{15}$  .

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[4]

17. (a) Find  $\int \left( 3x^4 + \sqrt{x} + \frac{1}{x^2} \right) dx$ .

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(b) Showing all your working, evaluate  $\int_1^2 (x^3 + 2) dx$ .

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**WJEC LEVEL 2 CERTIFICATE IN ADDITIONAL MATHEMATICS**

**MARK SCHEME**

	<b>WJEC Level 2 Certificate in Additional Mathematics Specimen Paper</b>	<b>Mark</b>	<b>Comments</b>
1	<p>(a)(i) <math>\frac{125}{81}</math></p> <p>(ii) Sight of <math>16^{-1}</math> or <math>2^{-4}</math> or <math>1/(2^4)</math> AND <math>1/16</math></p> <p>(b) (i) <math>\frac{20y^{\frac{3}{4}}}{y^{\frac{3}{4}}}</math> <math>20</math></p> <p>(ii) <math>\frac{x^{\frac{1}{3}}(2+5x)}{4x^{\frac{1}{3}}}</math> <math>\frac{2+5x}{4}</math></p>	<p>B2</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>7</p>	<p>B1 for either <math>125</math> or <math>\frac{1}{81}</math> or <math>\frac{5^3}{3^4}</math> <i>Answer only, no working shown, B0</i></p> <p><i>Answer only, no working shown, B0</i></p> <p>CAO</p> <p>CAO</p>
2	<p>(a) <math>3(-2)^3 - (-2)^2 + 5(-2) + 42</math> <math>= 4</math></p> <p>(b)(i) Substitute <math>x = 4</math> showing <math>= 0</math></p> <p>(ii) <math>(x-4)(2x^2 + bx + c)</math> or intention to divide by <math>(x-4)</math> with <math>2x^2</math> shown <math>(x-4)(2x^2 + 5x - 3)</math> <math>(x-4)(2x-1)(x+3)</math></p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A2</p> <p>A1</p> <p>8</p>	<p>Or division method giving <math>3x^2 - 7x \dots</math></p> <p>Or division method giving <math>2x^2 + 5x \dots</math></p> <p>A1 for <math>+5x</math> or <math>-3</math>. Or use of factor theorem A1 <math>(x+3)</math>, A1 <math>(2x-1)</math> CAO. Penalise further working. <i>If no marks B1 for <math>(x+3)</math> or <math>(2x-1)</math></i></p>
3	<p>(a) <math>35x^4 + 1 (+0)</math></p> <p>(b) <math>-6x^{-7}</math></p> <p>(c) <math>\frac{2}{3}x^{-1/3}</math></p>	<p>B3</p> <p>B1</p> <p>B1</p> <p>5</p>	<p>B1 for each term. Accept <math>5 \times 7</math> as 35</p> <p>Index needs to be simplified</p>
4	<p><math>\tan 30 = 5/BF</math> or <math>\tan 45 = 5/FC</math> or <math>FC = 5</math> Sight of <math>\tan 30 = 1/\sqrt{3}</math> <math>(BF + FC =) 5/\tan 30 + 5/\tan 45</math> <math>5\sqrt{3} + 5 (= 5(\sqrt{3} + 1))</math></p>	<p>M1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>4</p>	<p>(<math>F</math> is the foot of the perpendicular from <math>A</math>)</p> <p>OR equivalent, <math>5/\tan 30 + 5</math> Convincing</p>
5	<p><math>(dy/dx =) 3x^2 - 3</math> <math>dy/dx = 0</math> or <math>3x^2 - 3 = 0</math> <math>x = 1</math> or <math>x = -1</math> <math>y = -4</math> or <math>y = 0</math></p> <p><math>d^2y/dx^2 = 6x</math></p> <p><math>(-1, 0): d^2y/dx^2 &lt; 0</math>, point is a maximum <math>(1, -4): d^2y/dx^2 &gt; 0</math>, point is a minimum</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>7</p>	<p>FT their <math>dy/dx</math> form <math>ax^2 + b</math></p> <p>FT their <math>x</math> substitution <i>Answer only, no working shown, M0 A0 A0</i> Or first derivative test, interpretation of first derivative test. Or alternative.</p>
6	<p><math>x^2 + xy = 198</math> <math>6x + 2y = 80</math> or <math>3x + y = 40</math> <math>x^2 + x(40 - 3x) = 198</math> <math>2x^2 - 40x + 198 = 0</math> or <math>x^2 - 20x + 99 = 0</math> <math>(x-9)(x-11) = 0</math> or equivalent <math>x = 9</math> (or 11) Other length 13 (cm)</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>7</p>	<p>FT for their equations CAO or negative of either quadratic Factorising their quadratic or formula method CAO FT their <math>x</math> or <math>y</math> value for shortest side logic</p>

	<p align="center"><b>WJEC Level 2 Certificate in Additional Mathematics Specimen Paper</b></p>	<p align="center"><b>Mark</b></p>	<p align="center"><b>Comments</b></p>
7	$\int (2x - x^2) dx$ $= x^2 - x^3/3$ <p>Use of correct limits in order</p> <p align="right">4/3</p>	<p>M1 A2</p> <p>m1 A1</p> <p align="center">5</p>	<p>Intention to integrate Do not penalise dx omitted. Limits not required A1 for each</p> <p>CAO <i>No marks for use of trapezium rule</i></p>
8	<p>Attempt to clear fractions</p> $2(x - 2) + 4(2x + 3)(x - 2) = 4x(2x + 3)$ $- 28 = 14x$ $x = -2$	<p>M1 A1 A1 A1</p> <p align="center">4</p>	<p>For initial correct idea, including expressing all terms over common denominators (allow a slip) FT the one slip</p>
9	<p>(a) <math>(4x - 3)(3x + 5)</math> <math>\frac{3}{4}</math> or <math>-5/3</math></p> <p>(b) <math>(x + 8)^2 \pm \dots</math> ..... <math>-64 (+ 3)</math> Least value <math>-61</math></p>	<p>B2 B2 B1 B1 B1</p> <p align="center">7</p>	<p>B1 for <math>(4x - 3)</math>, B1 for <math>(3x + 5)</math> FT for their factors. B1 for each answer Sight of <math>(x + 8)^2</math> Sight of <math>-64</math>, or implied (e.g by = 64) CAO</p>
10	<p>Area square base = <math>x^2</math> Area triang. side = <math>\frac{1}{2}x^2 \sin 60</math> or <math>\frac{1}{2}x\sqrt{(x^2 - (x/2)^2)}</math></p> $x^2/2 \cdot \sqrt{3}/2 \text{ or } \frac{1}{2}x\sqrt{(3x^2/4)}$ <p>Total surface area = <math>x^2 + \frac{4(x^2\sqrt{3})}{4}</math> <math>= x^2(1 + \sqrt{3})</math></p>	<p>B1 M1</p> <p>A1 B1</p> <p>A1 5</p>	<p>Or equivalent, e.g. tan to find height, <math>\tan 60 \cdot x/2</math> followed by <math>\frac{1}{2}x \cdot \tan 60 \cdot x/2</math> Or equivalent, e.g. <math>\frac{1}{2}x \cdot \sqrt{3} \cdot x/2</math> FT their <math>x^2 + 4 \times</math> area of triangular side</p> <p>CAO</p>
11	<p>Attempt <math>dy/dx</math>, one term correct <math>dy/dx = 3x^2 - 6x</math> at <math>x = -1</math> gradient = 9 when <math>x = -1</math> <math>y = -2</math> Equation <math>(y - -2) = 9(x - -1)</math></p> $y + 2 = 9(x + 1) \text{ ISW } (y = 9x + 7)$	<p>M1 A1 A1 B1 m1</p> <p>A1 6</p>	<p>FT equivalent level of difficulty</p> <p>Or alternative method of setting up the equation FT their value of gradient &amp; point only if M1 awarded. Depends on use of calculus CAO. Any form</p>
12	<p>(a) <math>PQ^2 = (14 - 2)^2 + (19 - 3)^2 (=12^2 + 16^2)</math> <math>PQ = \sqrt{400} (=20)</math></p> <p>(b) Grad. PQ <math>(19 - 3) / (14 - 2)</math> <math>= 16/12</math> Grad. perpendicular <math>-12/16</math></p>	<p>M1 A1</p> <p>M1 A1 B1</p> <p align="center">5</p>	<p>Allow 1 slip or error CAO</p> <p>Ignore incorrect cancelling throughout (b) FT <math>-1/\text{grad PQ}</math>. Do not accept fraction of fraction</p>
13	<p>(a) <math>y + \delta y = (x + \delta x)^2 - (x + \delta x)</math> Intention to subtract <math>(y =) x^2 - x</math> to find <math>\delta y</math> <math>\delta y = 2x\delta x + (\delta x)^2 - \delta x</math> <math>\delta y/\delta x = 2x + \delta x - 1</math> and <math>\lim_{\delta x \rightarrow 0} dy/dx = 2x - 1</math></p> <p>(b) <math>2x - 1 = 3</math> <math>x = 2</math></p>	<p>M1 m1 A1 M1 A1</p> <p>M1 A1 7</p>	<p>Or alternative notation. Allow if final bracket omitted</p> <p>Accept <math>\delta x^2</math> as meaning <math>(\delta x)^2</math> FT equivalent level of difficulty CAO. Must follow from correct working <i>Use of <math>dy/dx</math> throughout max 4 marks only, final A0</i></p> <p>FT from their response in (a)</p>



	<p align="center"><b>WJEC Level 2 Certificate in Additional Mathematics Specimen Paper</b></p>	<p align="center"><b>Mark</b></p>	<p align="center"><b>Comments</b></p>
14	(a) General sine curve through (0,0), y values $\pm 2$ Period clearly $120^\circ$ (b) $70^\circ, 110^\circ$ only	B1 B1 B3  5	B2 for any 1 correct, B1 for indication of 2 values on their graph or sight of $-10^\circ$ or $210^\circ$ or $330^\circ$
15	Grad. given line = $-4$ so perpendicular grad. = $\frac{1}{4}$ Equation $y = \frac{1}{4}x$ OR $4y = x$  Clues needed 1 and 3	B1 B1  B1  QWC2          5	FT $-1$ /their gradient, or their perpendicular gradient (with slip) with $c = 0$ Implied in working or embedded in strategy  QWC2 Presents material in a coherent and logical manner, using acceptable mathematical form, and with few if any errors in spelling, punctuation and grammar.  QWC1 Presents material in a coherent and logical manner but with some errors in use of mathematical form, spelling, punctuation or grammar OR evident weaknesses in organisation of material but using acceptable mathematical form, with few if any errors in spelling, punctuation and grammar.  QWC0 Evident weaknesses in organisation of material, and errors in use of mathematical form, spelling, punctuation and grammar.
16	$\{6(2x) + 5(x-1) + 3(3x+5)\}/30$ $\{12x + 5x - 5 + 9x + 15\}/30 = \{26x + 10\}/30$ <b>and</b> $(13x + 5)/15$ or showing LHS $\equiv$ RHS	M1 A1 A2  4	Attempt to use common denominator Or equivalent (e.g. all/60) A1 for 1 slip or no conclusion
17	(a) $\frac{3}{5}x^5 + \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{x}$ <p align="right">+ c (constant)</p> (b) $\frac{x^4}{4} + 2x$ $\left[\frac{x^4}{4} + 2x\right]_1^2$ $\left(\frac{2^4}{4} + 2(2)\right) - \left(\frac{1^4}{4} + 2(1)\right)$ $= \frac{23}{4} \quad (=5\frac{3}{4})$	B3 B1  B2  M1  m1  A1  9	B1 for each term (Accept unsimplified $+ - x^{-1}$ ISW)  B1 for $\frac{x^4}{4}$ or $2x$  FT their integration. Intention to use 2, 1 and subtract  FT for correct use of limits  CAO, not FT Answer only, no working, M0 m0 A0

**ASSESSMENT GRID**

**WJEC LEVEL 2 CERTIFICATE IN ADDITIONAL MATHEMATICS**

	Assessment Objectives (Marks)			Total Mark	QWC
	AO1 (60% - 70%)	AO2 (15% - 25%)	AO3 (10% - 20%)		
Question					
1	7			7	
2	8			8	
3	5			5	
4		4		4	
5	7			7	
6			7	7	
7	5			5	
8	4			4	
9	7			7	
10			5	5	
11		6		6	
12	5			5	
13	5	2		7	
14	2	3		5	
15			5	5	✓
16	4			4	
17	9			9	
<b>Totals</b>	<b>68</b>	<b>15</b>	<b>17</b>	<b>100</b>	