

GCE AS/A LEVEL



WJEC GCE AS/A Level in MATHEMATICS

APPROVED BY QUALIFICATIONS WALES

GUIDANCE FOR TEACHING

Teaching from 2017

This Qualifications Wales regulated qualification is not available to centres in England.



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INTRODUCTION

The **WJEC AS and A Level Mathematics** qualifications, approved by Qualification Wales for first teaching from September 2017, are available to:

- All schools and colleges in Wales
- Schools and colleges in independent regions such as Northern Ireland, Isle of Man and the Channel Islands

The AS will be awarded for the first time in Summer 2018, using grades A – E; the A level will be awarded for the first time in Summer 2018, using grades A* – E.

The qualification provides a broad, coherent, satisfying and worthwhile course of study. It encourages learners to develop confidence in, and a positive attitude towards, mathematics and to recognise its importance in their own lives and to society.

The specification builds on the tradition and reputation WJEC has established for clear, reliable assessment supported by straightforward, accessible guidance and administration.

In addition to this guide, support is provided in the following ways:

- Specimen assessment materials
- Face-to-face CPD events
- Examiners' reports on each question paper
- Free access to past question papers and mark schemes via the secure website
- Direct access to the subject officer
- Free online resources
- Exam Results Analysis
- Online Examination Review

AIMS OF THE GUIDANCE FOR TEACHING

The principal aims of this Guidance for Teaching are to offer support to teachers in delivery of the new WJEC GCE AS/A Level Mathematics specification and to offer guidance on the requirements of the qualification and the assessment process.

The guide is **not intended as a comprehensive reference**, but as support for professional teachers to develop stimulating and exciting courses tailored to the needs and skills of their own students in their particular institutions.

The guide contains detailed clarification and guidance on the subject content for all units in the qualification.

The guide also contains a section on assessment objectives and how the different elements of these can be assessed in examination papers.

Written examination: 2 hours 30 minutes
 25% of A level qualification (62.5% of AS qualification)
 120 marks

Topics	Guidance
2.1.1 Proof	
<p>Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including</p> <ul style="list-style-type: none"> (a) proof by deduction, (b) proof by exhaustion, (c) disproof by counter example. 	<p>Proof by deduction to include the proofs of the laws of logarithms and differentiation from first principles for small positive powers of x.</p> <p>Learners will also be expected to generate their own proofs by applying the structure and techniques associated with standard mathematical proofs.</p>

Topics	Guidance
2.1.2 Algebra and Functions	
<p>Understand and use the laws of indices for all rational exponents.</p> <p>Use and manipulate surds, including rationalising the denominator.</p>	<p>Know the rules of indices:</p> $a^p \times a^q = a^{p+q}, \quad a^p \div a^q = a^{p-q}, \quad (a^p)^q = a^{pq},$ $a^{-n} = \frac{1}{a^n}, \quad (\sqrt[q]{a})^p = a^{\frac{p}{q}} \quad \text{and} \quad a^0 = 1.$ <p>To include rationalising fractions such as $\frac{2+3\sqrt{5}}{3-2\sqrt{5}}$ and $\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}$.</p> <p>Learners will be expected to know and use the results:</p> $\sqrt{ab} = \sqrt{a} \times \sqrt{b}, \quad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}, \quad (\sqrt{a})^2 = a.$ <p>Learners will be expected to give answers in their simplest form, eg. cancelling common factors in the numerator and denominator,</p> $\frac{6+4\sqrt{5}}{2} = 3+2\sqrt{5}$

Topics	Guidance
<p>Work with quadratic functions and their graphs. The discriminant of a quadratic function, including the conditions for real roots and repeated roots.</p> <p>Completing the square.</p> <p>Solution of quadratic equations in a function of the unknown.</p>	<p>The nature of the roots of a quadratic equation. Learners will be expected to understand and use the following conditions:</p> $b^2 - 4ac > 0 \quad 2 \text{ real and distinct roots}$ $b^2 - 4ac = 0 \quad 2 \text{ equal real roots / 1 repeated real root}$ $b^2 - 4ac < 0 \quad \text{no real roots}$ <p>Learners should be able to complete the square of a polynomial of the form $ax^2 + bx + c$, where $a \neq 1$. To include finding the maximum or minimum value of a quadratic function, eg. $y = 2(x - 3)^2 + 4$ has a minimum at (3,4).</p> <p>To include by factorisation, use of the formula and completing the square. Unknowns include powers of x, trigonometric, exponential or logarithmic functions, eg. $x^4 - 2x^2 + 1 = 0$</p>
<p>Solve simultaneous equations in two variables by elimination and by substitution, including one linear and one quadratic equation.</p>	<p>To include finding the points of intersection or the point of contact of a line and a curve. Eg. Solve</p> $\begin{array}{l} 2x = y - 1 \\ x^2 - 3y + 11 = 0 \end{array} \quad \text{or} \quad \begin{array}{l} y^2 + xy = 3 \\ 2x + y = 1 \end{array} \quad \text{or} \quad \begin{array}{l} x^2 + 4y^2 = 2 \\ 2y + x + 2 = 0 \end{array}$

Topics	Guidance
<p>Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically, including inequalities with brackets and fractions.</p> <p>Express solutions through the correct use of 'and' and 'or', or through set notation.</p> <p>Represent linear and quadratic inequalities graphically.</p>	<p>To include the solution of inequalities such as $1 - 2x < 4x + 7$, $\frac{x}{2} \geq 2(1 - 3x)$ and $x^2 - 6x + 8 \geq 0$.</p> <p>To include: $x < a$ or $x < b = \{x : x < a\} \cup \{x : x < b\}$ $x > c$ and $x < d = \{x : x > c\} \cap \{x : x < d\}$</p> <p>To include, for example, $y > x + 1$ (a strict inequality) and $y \geq ax^2 + bx + c$ (a non-strict inequality).</p>
<p>Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the Factor Theorem.</p>	<p>The use of the Factor Theorem will be restricted to cubic polynomials and the solution of cubic equations.</p> <p>Learners may be required to factorise expressions such as $x^3 + 2x^2 - x - 2$ and $x^3 - 7x - 6$.</p> <p>Learners should know that if</p> <ul style="list-style-type: none"> • $f(c) = 0$ then $(x - c)$ is a factor of $f(x)$ • $f\left(\frac{c}{a}\right) = 0$ then $(ax - c)$ is a factor of $f(x)$

Topics	Guidance
<p>Understand and use graphs of functions; sketch curves defined by simple equations, including polynomials.</p> <p>$y = \frac{a}{x}$ and $y = \frac{a}{x^2}$, including their vertical and horizontal asymptotes.</p> <p>Interpret algebraic solutions of equations graphically. Use intersection points of graphs of curves to solve equations.</p> <p>Understand and use proportional relationships and their graphs.</p>	<p>The equations will be restricted to the form $y = f(x)$. Learners may be required to sketch simple cubic functions, including those with repeated roots, eg. $y = x(x - 2)^2$</p> <p>Eg. The solution of $x^2 + 4y^2 = 2$ and $x + 2y = 2$ is $x = 1, y = 0.5$ (ie. only one point of intersection), therefore, $x + 2y = 2$ must be a tangent to the curve $x^2 + 4y^2 = 2$.</p> <p>Use of the proportional symbol \propto or equation with an unknown constant, eg. $y = kx$, $y = kx^2$, $y = \frac{k}{x}$.</p>
<p>Understand the effect of simple transformations on the graph of $y = f(x)$ including sketching associated graphs: $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$.</p>	<p>To include describing the transformation of a function from given graphs. Only single transformations will be considered.</p>

Topics	Guidance
<p>2.1.3 Coordinate geometry in the (x, y) plane</p>	
<p>Understand and use the equation of a straight line, including the forms $y = mx + c$, $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$; gradient conditions for two straight lines to be parallel or perpendicular.</p> <p>Be able to use straight line models in a variety of contexts.</p>	<p>To include:</p> <ul style="list-style-type: none"> finding the gradient, equation, length and midpoint of a line joining two given points; the condition for two lines to be parallel ($m = m'$) or perpendicular $\left(m' = -\frac{1}{m}\right)$ finding the equations of lines which are parallel or perpendicular to a given line; finding the point of intersection of two lines. <p>Eg. velocity against time for constant acceleration.</p>
<p>Understand and use the coordinate geometry of the circle using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$; completing the square to find the centre and radius of a circle.</p> <p>Use of the following circle properties:</p> <ol style="list-style-type: none"> the angle in a semicircle is a right angle; the perpendicular from the centre to a chord bisects the chord; the radius of a circle at a given point on its circumference is perpendicular to the tangent to the circle at that point. 	<p>To also be familiar with the equation of a circle in the form $x^2 + y^2 + 2gx + 2fy + c = 0$.</p> <p>To include:</p> <ul style="list-style-type: none"> finding the equations of tangents, the condition for two circles to touch internally or externally, finding the points of intersection or the point of contact of a line and a circle.

Topics	Guidance
2.1.4 Sequences and Series - The Binomial Theorem	
<p>Understand and use the binomial expansion of $(a + bx)^n$ for positive integer n.</p> <p>The notations $n!$, $\binom{n}{r}$ and nCr.</p> <p>Link to binomial probabilities.</p>	<p>Eg. Find the coefficient of the x^3 term in the expansion of $(3 - 2x)^5$</p> <p>Eg. Find the greatest coefficient in the expansion of $(3x + 5)^9$.</p> <p>To include use of Pascal's triangle.</p>

Topics	Guidance
2.1.5 Trigonometry	
Understand and use the definitions of sine, cosine and tangent for all arguments.	Use of the exact values of the sine, cosine and tangent of 30° , 45° and 60° .
Understand and use the sine and cosine rules, and the area of a triangle in the form $\frac{1}{2} ab \sin C$.	To include the use of the sine rule in the ambiguous case. Sine rule: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$
Understand and use the sine, cosine and tangent functions. Understand and use their graphs, symmetries and periodicity.	Use of graphs to understand <ul style="list-style-type: none"> $\cos \theta = \cos(360^\circ - \theta)$ and $\cos(-\theta) = \cos \theta$ $\sin \theta = \sin(180^\circ - \theta)$ and $\sin(-\theta) = -\sin \theta$ $\tan \theta = \tan(180^\circ + \theta)$ and $\tan(-\theta) = -\tan \theta$
Understand and use $\tan \theta = \frac{\sin \theta}{\cos \theta}$. Understand and use $\cos^2 \theta + \sin^2 \theta = 1$.	These identities may be used to solve trigonometric equations or prove trigonometric identities.
Solve simple trigonometric equations in a given interval, including quadratic equations in \sin , \cos and \tan , and equations involving multiples of the unknown angle.	To include the solution of equations such as <ul style="list-style-type: none"> $3 \sin \theta = 1$, for $0^\circ \leq \theta \leq 360^\circ$ $\tan \theta = \frac{\sqrt{3}}{2}$, for $-180^\circ \leq \theta \leq 180^\circ$ $3 \cos 2\theta = -1$, for $0^\circ \leq \theta \leq 180^\circ$ $2 \cos^2 \theta + \sin \theta - 1 = 0$, for $0^\circ \leq \theta < 360^\circ$ $3 \sin^2 \theta + 5 \cos \theta - 5 = 0$, for $0^\circ < \theta \leq 360^\circ$

Topics	Guidance
2.1.6 Exponentials and logarithms	
<p>Know and use the function a^x and its graph, where a is positive.</p> <p>Know and use the function e^x and its graph.</p>	<p>Know that all graphs of the form $y = a^x$, where a is positive, pass through the point $(0,1)$.</p>
<p>Know that the gradient of e^{kx} is equal to ke^{kx} and hence understand why the exponential model is suitable in many applications.</p>	<p>Learners should be familiar with the basic properties of the exponential model, e.g. the ratios of consecutive terms are equal. Realise that when the rate of change is proportional to the y value, ie. $\frac{dy}{dx} \propto y$, an exponential model should be used.</p>
<p>Know and use the definition of $\log_a x$ as the inverse of a^x, where a is positive and $x \geq 0$.</p> <p>Know and use the function $\ln x$ and its graph.</p> <p>Know and use $\ln x$ as the inverse function of e^x.</p>	<p>Learners should be able to convert from index to logarithmic form and vice versa, ie. $p = q^r \Leftrightarrow r = \log_q p$.</p> <p>Know that $\log_c c = 1$ and $\log_c 1 = 0$.</p> <p>For example:</p> <ul style="list-style-type: none"> Given $y = 4$ when $x = 3$, find the value of k in the equation $y = e^{kx}$ Solve $e^{2x-1} = 3$ Solve $\ln(4x+3) = 2$

Topics	Guidance
<p>Understand and use the laws of logarithms.</p> $\log_a x + \log_a y = \log_a (xy)$ $\log_a x - \log_a y = \log_a \left(\frac{x}{y} \right)$ $k \log_a x = \log_a (x^k) \quad (\text{including, for example } k = -1, k = -1/2)$	<p>To include the proof of the laws of logarithms. Use of the laws of logarithms to solve equations and to simplify expressions involving logarithms. e.g. Simplify $\log_2 36 - 2\log_2 15 + \log_2 100 + 1$.</p> <p>Change of base will not be required.</p>
<p>Solve equations in the form $a^x = b$.</p>	<p>The use of a calculator to solve equations such as</p> <ul style="list-style-type: none"> (i) $3^x = 2$, (ii) $25^x - 4 \times 5^x + 3 = 0$. (iii) $4^{2x+1} = 5^x$
<p>Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for x and y.</p>	<p>Link to laws of logarithms. Understand that on a graph of $\log y$ against $\log x$, the gradient is n and the intercept is $\log a$, and that on a graph of $\log y$ against x, the gradient is $\log b$ and the intercept is $\log k$.</p>
<p>Understand and use exponential growth and decay; use in modelling (examples may include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as model for population growth.)</p> <p>Consideration of limitations and refinements of exponential models.</p>	<p>The formal differentiation and integration of formulae involving e^x and/or a^x will not be required. Be familiar with the meaning of the constants used in the model and to find values for them given appropriate conditions. Explore the behaviour of the variables for large values of t. Comment on the appropriateness of predicted values using the model. The formula for continuously compounded interest is $P(t) = P_0 e^{rt}$, where P_0 is the original principal amount, r is the annual rate of interest and t is the length of time (in years) the interest is applied.</p>

Topics	Guidance
2.1.7 Differentiation	
<p>Understand and use the derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a general point (x, y); the gradient of the tangent as a limit; interpretation as a rate of change; sketching the gradient function for a given curve; second order derivatives.</p> <p>Differentiation from first principles for small positive integer powers of x.</p> <p>Understand and use the second derivative as the rate of change of gradient.</p>	<p>Learners should know that $\frac{dy}{dx}$ denotes the rate of change of y with respect to x.</p> <p>The notation $\frac{dy}{dx}$ or $f'(x)$ may be used for the first derivative and $\frac{d^2y}{dx^2}$ or $f''(x)$ for the second derivative.</p> <p>Knowledge of the chain rule is not required here.</p> <p>Up to and including power of 3. To include polynomials up to and including a maximum degree of 3. Eg. Differentiate from first principles $y = x^3 - 2$</p> <p>Learners should understand that $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$.</p> <p>Be able to use the second derivative to distinguish between maximum and minimum points, ie.</p> <p style="padding-left: 40px;">$f''(x) > 0$ implies a minimum and $f''(x) < 0$ implies a maximum</p> <p>for points where $f'(x) = 0$.</p>
Differentiate x^n for rational n , and related constant multiples, sums and differences.	To include polynomials. May require the removal of brackets eg $(3x^2 - 1)(x + 7)$.

Topics	Guidance
Apply differentiation to find gradients, tangents and normals, maxima and minima, and stationary points. Identify where functions are increasing or decreasing.	To include finding the equations of tangents and normals. The use of maxima and minima in simple optimisation problems. To include simple curve sketching. Use of the conditions $f'(x) > 0$ for an increasing function and $f'(x) < 0$ for a decreasing function.
2.1.8 Integration	
Know and use the Fundamental Theorem of Calculus.	Integration as the reverse process of differentiation. The constant of integration is required for indefinite integrals.
Integrate x^n (excluding $n = -1$) and related sums, differences and constant multiples.	To include polynomials and expressions such as $2x^2 - \sqrt{x}$, $\frac{x+1}{\sqrt{x}}$.
Evaluate definite integrals. Use a definite integral to find the area under a curve.	To include finding the area of a region between a straight line and a curve. The area may include a region below the x -axis.

Topics	Guidance
2.1.9 Vectors	
Use vectors in two dimensions.	To include the use of the unit vectors, i and j . Learners should be familiar with column vectors and the notation \overrightarrow{CD} and c .
Add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations.	To include the condition for two vectors to be parallel.
Understand and use position vectors; calculate the distance between points represented by position vectors. Use vectors to solve problems in pure mathematics.	Use of $\mathbf{AB} = \mathbf{b} - \mathbf{a}$ or $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$. To include the use of position vectors given in terms of unit vectors. To include the use and derivation of the position vector of a point dividing a line in a given ratio.

AS UNIT 2 – APPLIED MATHEMATICS A

Written examination: 1 hour 45 minutes
 15% of A level qualification (37.5% of AS qualification)
 75 marks

The paper will comprise two sections:

Section A: Statistics (40 marks)

Section B: Mechanics (35 marks)

Topics	Guidance
STATISTICS	
2.2.1 Statistical Sampling	
Understand and use the terms 'population' and 'sample'. Use samples to make informal inferences about the population.	
Understand and use sampling techniques, including simple random sampling, systematic sampling and opportunity sampling.	
Select or critique sampling techniques in the context of solving a statistical problem, including understanding that different samples can lead to different conclusions about the population.	

Topics	Guidance
2.2.2 Data presentation and interpretation	
<p>Interpret diagrams for single-variable data, including understanding that area in a histogram represents frequency.</p> <p>Connect to probability distributions.</p>	<p>Learners should be familiar with box and whisker diagrams and cumulative frequency diagrams and how to use them to compare sets of data.</p> <p>Qualitative assessment of skewness is expected and the use of the terms symmetric, positive skew or negative skew</p>
<p>Interpret scatter diagrams and regression lines for bivariate data, including recognition of scatter diagrams which include distinct sections of the population.</p> <p>(Calculations of coefficients of regression lines are excluded.)</p> <p>Understand informal interpretation of correlation.</p> <p>Understand that correlation does not imply causation.</p>	<p>Equations of regression lines may be given in a question and learners asked to make predictions using it.</p> <p>Learners should be familiar with the regression lines y on x, and x on y.</p> <p>Use of the terms positive, negative, zero, strong and weak is expected.</p>
<p>Interpret measures of central tendency and variation, extending to standard deviation.</p> <p>Be able to calculate standard deviation, including from summary statistics.</p>	<p>Measures of central tendency: mean, median, mode.</p> <p>Measures of central variation: variance, standard deviation, range, interquartile range.</p>
<p>Recognise and interpret possible outliers in data sets and statistical diagrams.</p> <p>Select or critique data presentation techniques in the context of a statistical problem.</p> <p>Be able to clean data, including dealing with missing data, errors and outliers.</p>	<p>Use of $Q_1 - 1.5 \times IQR$ and $Q_3 + 1.5 \times IQR$ to identify outliers.</p> <p>Other methods of identifying outliers may be used, but formulae will be given in the question.</p>

Topics	Guidance
2.2.3 Probability	
<p>Understand and use mutually exclusive and independent events when calculating probabilities.</p> <p>Link to discrete and continuous distributions.</p>	<p>To include the multiplication law for independent events: $P(A \cap B) = P(A)P(B)$.</p>
<p>Use Venn diagrams to calculate probabilities.</p>	<p>Use of set notation and associated language is expected.</p> <p>To include the generalised addition law: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.</p> <p>Conditional probability will not be assessed in this unit.</p>
2.2.4 Statistical distributions	
<p>Understand and use simple, discrete probability distributions.</p> <p>Understand and use,</p> <ul style="list-style-type: none"> • the binomial distribution, as a model • the Poisson distribution, as a model • the discrete uniform distribution, as a model <p>(Calculation of mean and variance of discrete random variables is excluded.)</p>	<p>Use of $\sum P(X = x) = 1$.</p> <p>To include using distributions to model real world situations and to comment on their appropriateness.</p>

Topics	Guidance
Calculate probabilities using <ul style="list-style-type: none"> • the binomial distribution. • the Poisson distribution. • the discrete uniform distribution. 	Use of the binomial formula and tables / calculator. Use of the Poisson formula and tables / calculator Use of the formula for the discrete uniform distribution.
Select an appropriate probability distribution for a context, with appropriate reasoning, including recognising when the binomial, Poisson or discrete uniform model may not be appropriate.	

Topics	Guidance
2.2.5 Statistical hypothesis testing	
<p>Understand and apply the language of statistical hypothesis testing, developed through a binomial model: null hypothesis, alternative hypothesis, significance level, test statistic, 1-tail test, 2-tail test, critical value, critical region, acceptance region, p-value.</p>	<p>The p-value is the probability that the observed result or a more extreme one will occur under the null hypothesis H_0. For uniformity, interpretations of a p-value should be along the following lines:</p> <p>$p < 0.01$; there is very strong evidence for rejecting H_0. $0.01 \leq p \leq 0.05$; there is strong evidence for rejecting H_0. $p > 0.05$; there is insufficient evidence for rejecting H_0.</p>
<p>Conduct a statistical hypothesis test for the proportion in the binomial distribution and interpret the results in context.</p> <p>Understand that a sample is being used to make an inference about the population and appreciate that the significance level is the probability of incorrectly rejecting the null hypothesis.</p>	<p>Conclusions should be stated in a way as to reflect the fact that they are not certain.</p> <p>For example, "There is strong evidence to reject H_0. It is likely that the manufacturer's claim is correct." Or "There is insufficient evidence for rejecting H_0. There is no reason to suspect that there has been an increase in sales following the manager's changes." An example of an incorrect conclusion is as follows: "Reject H_0. There are now more red cars." Stated correctly this would be: "There is strong evidence to reject H_0. It is likely that there has been an increase in the number of red cars using the given car park."</p>
<p>Interpret and calculate Type I and Type II errors, and know their practical meaning.</p>	

Topics	Guidance
MECHANICS	
2.2.6 Quantities and units in mechanics	
Understand and use fundamental quantities and units in the S.I. system; length, time and mass.	S.I. base units: metres (m), seconds (s), kilograms (kg). Convert between S.I. units; for example, g to kg, cm to m.
Understand and use derived quantities and units: velocity, acceleration, force, weight.	<p>Derived quantities</p> <ul style="list-style-type: none"> • velocity (m s^{-1} or m/s), • acceleration (m s^{-2} or m/s^2), • force/weight (N). <p>Understand that weight is a force where, $W = mg$ (N).</p> <p>Learners should be able to state the appropriate unit(s) alongside a quantity where requested.</p> <p>Conversions: km h^{-1} to m s^{-1} and vice versa</p>
2.2.7 Kinematics	
Understand and use the language of kinematics: position, displacement, distance travelled, velocity, speed, acceleration.	<p>Learners should be able to distinguish between</p> <ul style="list-style-type: none"> • displacement and distance travelled; • velocity and speed; • acceleration and magnitude of acceleration. <p>Retardation or deceleration may be used to describe negative acceleration.</p> <p>Use of</p> $\text{Average speed/velocity} = \frac{\text{total distance/displacement}}{\text{total time}}$

Topics	Guidance
<p>Understand, use and interpret graphs in kinematics for motion in a straight line: displacement against time and interpretation of the gradient; velocity against time and interpretation of the gradient and the area under the graph</p>	<p>Learners may be expected to sketch displacement-time and velocity-time graphs. Graphs may include negative displacements/velocities. Axes should be clearly labelled with appropriate units.</p>
<p>Understand, use and derive the formulae for constant acceleration for motion in a straight line.</p>	<p>Learners should be able to recall and use the following formulae:</p> <ul style="list-style-type: none"> • $v = u + at$ • $s = ut + \frac{1}{2}at^2$ • $s = \frac{1}{2}(u + v)t$ • $v^2 = u^2 + 2as$ <p>Candidates may use $s = vt - \frac{1}{2}at^2$.</p> <p>Derivation of these formulae may require:</p> <ul style="list-style-type: none"> • Substitution • Use and interpretation of a graph • Use of calculus (see 'use calculus in kinematics' section below) <p>To include vertical motion under gravity. Gravitational acceleration, g.</p> <p>The inverse square law for gravitation is not required and g may be assumed to be constant, but learners should be aware that g is not a universal constant but depends on location. The value 9.8 ms^{-2} can be used for the acceleration due to gravity, unless explicitly stated otherwise. For example, learners may be required to work algebraically in terms of g.</p>

Topics	Guidance
<p>Use calculus in kinematics for motion in a straight line.</p>	<p>To include the use of $v = \frac{dr}{dt}$, $a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$, $r = \int v dt$, $v = \int a dt$, where v, a and r are given in terms of t.</p> <p>The required calculus will be consistent with that covered in sections 2.1.7 and 2.1.8 of AS Unit 1.</p> <p>For modelling purposes, a function may be defined piecewise. For example,</p> $a = \begin{cases} 4 - 2t & 0 \leq t \leq 2 \\ 0 & t > 2 \end{cases}$ <p>Note that, in line with the above 'graphs in kinematics' section, learners may be required to interpret and/or sketch velocity/displacement against time graphs in cases where acceleration is not constant.</p> <p>The required sketching will be of a level consistent with that covered in section 2.1.2 of AS Unit 1.</p>

Topics	Guidance
2.2.8 Forces and Newton's laws	
<p>Understand the concept of a force. Understand and use Newton's first law.</p>	<p>Learners should be familiar with the terms: weight, tension, thrust or compression, normal reaction, friction, driving force. Including forces in 2-D vector form, e.g. $\mathbf{F} = 2\mathbf{i} - 3\mathbf{j}$.</p> <p>Learners only require a basic understanding of the concept of friction and they should know that friction will oppose the direction of motion. Coefficient of friction is not required at AS.</p> <p>Newton's first law: <i>A particle that is at rest (or moving with constant velocity) will remain at rest (or moving with constant velocity) until acted upon by an external force.</i></p>
<p>Understand and use Newton's second law for motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors).</p>	<p>Use of $F = ma$ or $\mathbf{F} = m\mathbf{a}$ for constant mass and constant force.</p> <p>Learners are not required to resolve forces at AS.</p>
<p>Understand and use weight and motion in a straight line under gravity; gravitational acceleration, g, and its value in S.I. units to varying degrees of accuracy.</p> <p>(The inverse square law for gravitation is not required and g may be assumed to be constant, but learners should be aware that g is not a universal constant but depends on location.)</p>	<p>Learners should be able to distinguish between mass and weight. Forces will be constant and will include weight, normal reaction, tension and thrust. To include problems involving lifts.</p> <p>The value 9.8 ms^{-2} can be used for the acceleration due to gravity, unless explicitly stated otherwise. For example, learners may be required to work algebraically in terms of g.</p>

Topics	Guidance
<p>Understand and use Newton's third law. Equilibrium of forces on a particle and motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors)</p> <p>Applications to problems involving smooth pulleys and connected particles.</p>	<p>Newton's third law: <i>Every action has an equal and opposite reaction.</i></p> <p>Learners are not required to resolve forces at AS.</p> <p>Problems involving particles connected by strings passing over smooth, fixed pulleys or pegs; one particle will be freely hanging and the other particle may be</p> <ul style="list-style-type: none"> (i) freely hanging, (ii) on a smooth, horizontal plane. <p>Problems may involve contexts such as a car towing a trailer. Learners should use the fact that a system in which none of its components have any relative motion may be modelled as a single particle with the mass of the whole system. For example, treating car and trailer as a single particle.</p>
<p>2.2.9 Vectors</p>	
<p>Calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form.</p>	<p>Resolving forces is not required at AS. Therefore, questions will only involve the conversion of horizontal/vertical forces into component form. For example,</p> <ul style="list-style-type: none"> • A particle moving due north with speed 6 m s^{-1} can be converted to $\mathbf{v} = 6\mathbf{j} \text{ m s}^{-1}$.
<p>Use vectors to solve problems in context, including forces.</p>	<p>Does not include kinematics problems. To include the use of simple bearings.</p>

Notes on modelling

Learners should be able to

- state any necessary modelling assumptions. For example,
 - objects can be modelled as particles,
 - strings will be light and inextensible,
 - smooth implies that friction can be ignored,
 - no external forces are present unless otherwise stated, eg. air resistance can be ignored.
- comment on the appropriateness/limitations of any modelling assumptions made and briefly describe the possible mathematical consequences of the assumptions not holding.

Written examination : 2 hours 30 minutes
 35% of A level qualification
 120 marks

Topics	Guidance
2.3.1 Proof	
Proof by contradiction (including proof of the irrationality of $\sqrt{2}$ and the infinity of primes, and application to unfamiliar proofs).	
2.3.2 Algebra and Functions	
Simplify rational expressions, including by factorising and cancelling and by algebraic division (by linear expressions only).	Eg. $\frac{x^2 - 4}{x^2 + x - 2}$, $\frac{x^2 + 2x + 1}{3x^2 + 12x + 9}$
Sketch curves defined by the modulus of a linear function.	Be able sketch graphs of the form $y = ax + b $. To include solving equations and inequalities involving the modulus function, eg. $ 2x + 3 = x$, $ 3x - 1 < 5$

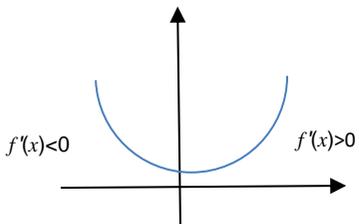
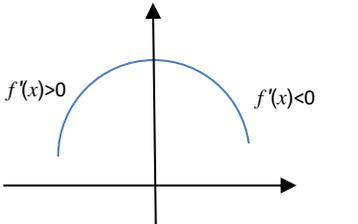
Topics	Guidance
<p>Understand and use composite functions; inverse functions and their graphs.</p>	<p>Understand and use the definition of a function. Understand and use the domain and range of functions.</p> <p>In the case of a function defined by a formula (with unspecified domain) the domain is taken to be the largest set such that the formula gives a unique image for each element of the set.</p> <p>Graph of $y = f^{-1}(x)$ is the image of the graph of $y = f(x)$ after reflection in the line $y = x$.</p> <p>The notation $f^{-1}(x)$ will be used for the inverse function and fg will be used for composition to mean $f(g(x))$.</p> <p>Know the condition for the inverse function to exist and be able to find the inverse function algebraically. Given the function $y = f(x)$, learners should be able to sketch the inverse function of $y = f(x)$ by reflection in the line $y = x$.</p>
<p>Understand the effect of combinations of transformations on the graph of $y = f(x)$, as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$ and $y = f(ax)$.</p>	<p>Learners should be able to apply combinations of these transformations to functions in the specification, eg e^x, x, $\cos x$ etc and sketch the resulting graphs.</p>
<p>Decompose rational functions into partial fractions (denominators not more complicated than squared linear terms and with no more than 3 terms, numerators constant or linear).</p>	<p>With denominators of the form $(ax + b)(cx + d)$, $(ax + b)(cx + d)(ex + f)$ and $(ax + b)(cx + d)^2$. Learners will not be expected to sketch the graphs of rational functions.</p>
<p>Use of functions in modelling, including consideration of limitations and refinements of the models.</p>	<p>Eg. modelling the behaviour of tides with trigonometric functions, exponential growth and decay, modelling volume and pressure of gases.</p>

Topics	Guidance
2.3.3 Coordinate geometry in the (x, y) plane	
Understand and use the parametric equations of curves and conversion between Cartesian and parametric forms.	To include finding the equations of tangents and normals to curves defined parametrically or implicitly. Eg. $x = \cos t$, $y = \sin t$ describes a circle with centre (0,0) and radius 1. Knowledge of the properties of curves other than the circle will not be expected.
Use parametric equations in modelling in a variety of contexts.	
2.3.4 Sequences and Series	
Understand and use the binomial expansion of $(a + bx)^n$, for any rational n , including its use for approximation. Be aware that the expansion is valid for $\left \frac{bx}{a} \right < 1$ (proof not required).	To include the expansion, in ascending powers of x , of expressions such as $(2 - x)^{\frac{1}{2}}$ and $\frac{(4 - x)^3}{(1 + 2x)}$ and be able to state the range of validity.
Work with sequences, including those given by a formula for the n th term and those generated by a simple relation of the form $x_{n+1} = f(x_n)$. Increasing sequences, decreasing sequences, periodic sequences.	To include being able to generate the terms of a sequence. Eg. $u_n = (-1)^n$ for $n > 0$ is a periodic function of order 2.
Understand and use sigma notation for sums of series.	Know the difference between a sequence and a series.

Topics	Guidance
Understand and work with arithmetic sequences and series, including the formulae for the n th term and the sum to n terms.	Use of $u_n = a + (n-1)d$. Use and proof of $S_n = \frac{n}{2}[2a + (n-1)d]$ and $S_n = \frac{n}{2}[a + l]$.
Understand and work with geometric sequences and series, including the formulae for the n th term and the sum of a finite geometric series. The sum to infinity of a convergent geometric series, including the use of $ r < 1$; modulus notation.	Use of $u_n = ar^{n-1}$. Use and proof of $S_n = \frac{a(1-r^n)}{1-r}$. Use of $S_\infty = \frac{a}{1-r}$ for $ r < 1$.
Use sequences and series in modelling.	Eg compound interest, loans.
2.3.5 Trigonometry	
Work with radian measure, including use for arc length, area of sector and area of segment.	Understand the relationship between degrees and radians. Use of degrees or radians in problems involving trig functions. Learners should know and be able to use the formulae $s = r\theta$ for arc length and $A = \frac{1}{2}r^2\theta$ for the area of a sector.
Understand and use the standard small angle approximations of sine, cosine and tangent. $\sin \theta \approx \theta$, $\cos \theta \approx 1 - \frac{\theta^2}{2}$ and $\tan \theta \approx \theta$, where θ is in radians.	Eg show $\frac{\cos^2 x - 1}{x \sin 2x} \approx -\frac{1}{2}$ for when x is small. Application to differentiation of $\sin x$ and $\cos x$ from first principles. Learners should know that $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ and $\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta} = 0$.

Topics	Guidance
Know and use exact values of \sin and \cos for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$ and multiples thereof, and exact values of \tan for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \pi$ and multiples thereof.	Eg Find the exact value of 165° .
Understand and use the definitions of \sec , cosec , \cot , \sin^{-1} , \cos^{-1} and \tan^{-1} . Understand the relationships of all of these to \sin , \cos and \tan and understand their graphs, ranges and domains.	For example, understand that $y = \sec x$ means $y = \frac{1}{\cos x}$ and $y = \cos^{-1} x$ means $\cos y = x$.
Understand and use $\sec^2 \theta \equiv 1 + \tan^2 \theta$ and $\operatorname{cosec}^2 \theta \equiv 1 + \cot^2 \theta$.	The solution of trigonometric equations such as $\sec^2 \theta + 5 = 5 \tan \theta$. May be used to prove trigonometric identities.
Understand and use double angle formulae. Use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$. Understand geometric proofs of these formulae.	Use of these formulae to solve equations in a given range, e.g. $\sin 2\theta = \sin \theta$, Applications to integration, e.g. $\int \cos^2 x \, dx$.
Understand and use expressions for $a \cos \theta + b \sin \theta$ in the equivalent forms of $r \cos(\theta \pm \alpha)$ or $r \sin(\theta \pm \alpha)$.	Use of these to solve equations in a given range, e.g. $3 \cos \theta + \sin \theta = 2$. Application to finding greatest and least values, e.g. the least value of $\frac{1}{3 \cos \theta + 4 \sin \theta + 10}$.
Construct proofs involving trigonometric functions and identities.	Eg. Prove that $\frac{\sin(A - B)}{\cos A \cos B} \equiv \tan A - \tan B$.

Topics	Guidance
2.3.6 Differentiation	
Differentiation from first principles for $\sin x$ and $\cos x$.	<p>Use of compound angle formulae and small angle approximations, ie. for $\cos x$, use of</p> $\frac{\cos(x + \delta x) - \cos x}{\delta x} = \frac{\cos x \cos \delta x - \sin x \sin \delta x - \cos x}{\delta x} \text{ and}$ <p>for $\sin x$, use of</p> $\frac{\sin(x + \delta x) - \sin x}{\delta x} = \frac{\sin x \cos \delta x + \cos x \sin \delta x - \sin x}{\delta x}$ <p>and let $\delta x \rightarrow 0$.</p> <p>Students may use δx or h.</p>

Topics	Guidance
<p>Understand and use the second derivative as the rate of change of gradient; connection to convex and concave sections of curves, and points of inflection.</p>	<p>Points of inflection to include stationary and non-stationary points. Use of the condition that:</p> <ul style="list-style-type: none"> if $f''(x) > 0$ in an interval, the function is convex in that interval; if $f''(x) < 0$ in an interval, the function is concave in that interval; <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <p>Convex</p>  <p>$f'(x)$ is increasing $f''(x) > 0$</p> </div> <div style="text-align: center;"> <p>Concave</p>  <p>$f'(x)$ is decreasing $f''(x) < 0$</p> </div> </div> <ul style="list-style-type: none"> at a point of inflection $f''(x)$ changes sign.
<p>Differentiate e^{kx}, a^{kx}, $\sin kx$, $\cos kx$, $\tan kx$, and related sums, differences and constant multiples.</p> <p>Understand and use the derivative of $\ln x$.</p>	<p>Use of the result $\frac{d}{dx}(a^{kx}) = ka^{kx} \ln a$.</p>
<p>Apply differentiation to find points of inflection.</p>	<p>Learners should know that if x is a point of inflection then $f''(x) = 0$ and the second derivative changes sign either side of x. In addition,</p> <ul style="list-style-type: none"> if $f'(x) = 0$, then the point of inflection is a stationary point; if $f'(x) \neq 0$, then the point of inflection is a non-stationary point.

Topics	Guidance
Differentiate using the product rule, the quotient rule and the chain rule, including problems involving connected rates of change and inverse functions.	To include the use of $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$ and $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$. To include differentiation of $\sec x$, $\operatorname{cosec} x$, $\cot x$, $\cos^{-1} x$, $\sin^{-1} x$, $\tan^{-1} x$.
Differentiate simple functions and relations defined implicitly or parametrically, for first derivative only.	To include finding equations of tangents and normal to curves given parametrically or implicitly. Eg. Find the equation of the normal to the curve $x^3 - 2xy^2 + y^3 = 5$ at the point (2, 1). Eg. The curve C has parametric equations $x = 2t$, $y = 5t^3$. Show that the equation of the tangent to C at the point P is $2y = 15p^2x - 20p^3$.
Construct simple differential equations in pure mathematics.	Only knowledge of first order differential equations will be required. Learners should be able to understand the terminology associated with proportionality (section 2.1.2 of AS Unit 1) and rates of change. For example, <ul style="list-style-type: none"> • directly/inversely proportional to; • rate of change, rate of increase/decrease.

Topics	Guidance
2.3.7 Integration	
Integrate e^{kx} , $\frac{1}{x}$, $\sin kx$, $\cos kx$ and related sums, differences and constant multiples.	<p>Use of the results:</p> <p>1) if $\int f(x)dx = F(x) + k$ then $\int f(ax + b)dx = \frac{1}{a}F(ax + b) + k$.</p> <p>2) $\int f'(g(x))g'(x)dx = f(g(x)) + c$</p> <p>To include the use of trigonometric identities such as double angle formulae to integrate functions such as $\cos^2 x$ and $\sin^2 2x$.</p>
Use a definite integral to find the area between two curves.	
Understand and use integration as the limit of a sum.	$\int_a^b f(x)dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(a + k\delta x)\delta x, \text{ where } \delta x = \frac{b-a}{n}$
<p>Carry out simple cases of integration by substitution and integration by parts.</p> <p>Understand these methods as the reverse processes of the chain rule and the product rule respectively.</p> <p>Integration by substitution includes finding a suitable substitution and is limited to cases where one substitution will lead to a function which can be integrated.</p> <p>Integration by parts includes more than one application of the method but excludes reduction formulae.</p>	<p>Use of $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$.</p> <p>Learners will be expected to integrate functions such as $(4 - 3x)^5$, $\frac{3}{(2x-1)^4}$, $x \cos x$.</p> <p>Eg. $\int x\sqrt{1+x^2} dx$, $\int \frac{x^3}{1+x^4} dx$, $\int \frac{\cos x}{\sin^3 x} dx$</p> <p>Eg. $\int x^2 e^x dx$, $\int e^{2x} \sin 3x dx$</p>

Topics	Guidance
Integrate using partial fractions that are linear in the denominator.	To include repeated linear factors. Eg. $\int \frac{5}{2+3x} dx$, $\int \frac{x-13}{(2+x)^2(2x-1)} dx$
Evaluate the analytical solution of simple first order differential equations with separable variables, including finding particular solutions. (Separation of variables may require factorisation involving a common factor.)	Questions will be set in pure mathematics only.
2.3.8 Numerical Methods	
Locate roots of $f(x) = 0$ by considering changes in sign of $f(x)$ in an interval of x in which $f(x)$ is sufficiently well-behaved. Understand how change of sign methods can fail.	Eg if the interval is too large so as to include an even number of roots, or if the function is discontinuous or has an asymptote.
Solve equations approximately using simple iterative methods; be able to draw associated cobweb and staircase diagrams. Solve equations using the Newton-Raphson method and other recurrence relations of the form $x_{n+1} = g(x_n)$. Understand how such methods can fail.	The iterative formula will be given. Consideration of the conditions for convergence will not be required. Understand convergence in geometrical terms by drawing cobweb and staircase diagrams. Newton-Raphson: If $x = a$ is an approximate solution of the equation $f(x) = 0$ then a better approximation is $x = b$ where $b = a - \frac{f(a)}{f'(a)}$. Method fails if the gradient is too small.

Topics	Guidance
<p>Understand and use numerical integration of functions, including the use of the trapezium rule and estimating the approximate area under a curve and limits that it must lie between.</p>	<p>Learners will be expected to use the trapezium rule to estimate the area under a curve and to determine whether it gives an overestimate or an underestimate of the area under a curve. May require the use of a sketched graph.</p> <p>Simpson's rule is excluded.</p>
<p>Use numerical methods to solve problems in context.</p>	<p>To solve problems in context which lead to equations that cannot be solved analytically.</p>

Written examination: 1 hour 45 minutes
 25% of A level qualification
 80 marks

The paper will comprise two sections:

Section A: Statistics (40 marks)

Section B: Differential Equations and Mechanics (40 marks)

Topics	Guidance
STATISTICS	
2.4.1 Probability	
Understand and use conditional probability, including the use of tree diagrams, Venn diagrams and two-way tables.	
Understand and use the conditional probability formula: $P(A \cap B) = P(A) P(B A) = P(B) P(A B)$.	
Modelling with probability, including critiquing assumptions made and the likely effect of more realistic assumptions.	

Topics	Guidance
2.4.2 Statistical distributions	
<p>Understand and use the continuous uniform distribution and Normal distributions as models.</p> <p>Find probabilities using the Normal distribution.</p> <p>Link to histograms, mean, standard deviation, points of inflection and the binomial distribution.</p>	<p>Calculation of the mean and variance of the uniform distribution is required.</p> <p>Use of calculator / tables to find probabilities. Linear interpolation in tables will not be required. The Normal approximation to the binomial distribution is not required.</p>
<p>Select an appropriate probability distribution for a context, with appropriate reasoning, including recognising when the continuous uniform or Normal model may not be appropriate.</p>	<p>The distributions from which the selection can be made are: Discrete: binomial, Poisson, uniform Continuous: Normal, uniform</p>
2.4.3 Statistical hypothesis testing	
<p>Understand and apply statistical hypothesis testing to correlation coefficients as measures of how close data points lie to a straight line and be able to interpret a given correlation coefficient using a given p-value or critical value.</p> <p>(The calculation of correlation coefficients is excluded.)</p>	<p>Learners will be expected to state hypotheses in terms of ρ, where ρ represents the population correlation coefficient.</p> <p>When using Pearson's product moment correlation coefficient in a hypothesis test, the data may be assumed to come from a bivariate normal distribution.</p>
<p>Conduct a statistical hypothesis test for the mean of a Normal distribution with known, given or assumed variance, and interpret the results in context.</p>	<p>Learners should know and be able to use the result that</p> $\text{if } X \sim N(\mu, \sigma^2) \quad \text{then} \quad \bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ <p>(The proof is excluded.)</p>

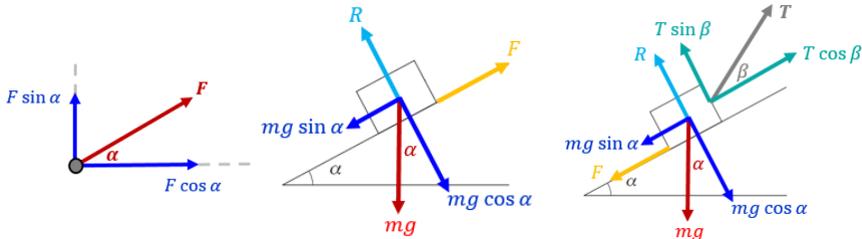
Topics	Guidance
DIFFERENTIAL EQUATIONS AND MECHANICS	
2.4.4 Trigonometry	
<p>Use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces.</p>	<p>Contexts may include, for example, wave motion as well as problems in vector form which involve resolving directions and quantities in mechanics into perpendicular components.</p> <p>For example, acceleration/velocity as a function of t,</p> <ul style="list-style-type: none"> • $a = 12 \sin(3t) - 8 \cos(2t)$ • $\mathbf{v} = \sin(3t)\mathbf{i} + 2 \cos(5t)\mathbf{j} + 3t^3 \mathbf{k}$

Topics	Guidance
2.4.5 Differentiation	
<p>Construct simple differential equations in context (contexts may include kinematics, population growth and modelling the relationship between price and demand).</p>	<p>Extension of section 2.3.6 of A2 Unit 3, to include Mechanics contexts as well as exponential growth and decay.</p> <p>Learners should be able to understand the terminology associated with proportionality (section 2.1.2 of AS Unit 1) and rates of change.</p> <p>For example,</p> <ul style="list-style-type: none"> • directly/inversely proportional to; • rate of change, rate of increase/decrease. <p>For example, the rate of change of a population is inversely proportional to the square of the size of the population P, i.e.</p> $\frac{dP}{dt} = \frac{k}{P^2}$ <p>To include setting up differential equations using $F = m \frac{dv}{dt}$.</p>

Topics	Guidance
2.4.6 Integration	
<p>Evaluate the analytical solution of simple first order differential equations with separable variables, including finding particular solutions.</p>	<p>Extension of section 2.3.7 of A2 Unit 3 to include contexts outlined in section 2.4.5 above. For example,</p> <ul style="list-style-type: none"> • $\frac{dv}{dt} = \frac{2t + \sec^2 t}{2v}$ • $m \frac{dv}{dt} + kv = mg$ • $\frac{dy}{dt} = \frac{1}{y} \cos(2t)$ <p>Separation of variables may require factorisation involving a common factor. For example,</p> <ul style="list-style-type: none"> • $\frac{dy}{dx} = xy + 2x$
<p>Interpret the solution of a differential equation in the context of solving a problem, including identifying limitations of the solution; includes links to kinematics.</p>	<p>If differential equations are used to model a situation, learners should be able to identify limitations of the model and/or suggest possible refinements to the model (see notes on modelling).</p>

Topics	Guidance
2.4.7 Quantities and units in mechanics	
Understand and use derived quantities and units for moments.	Include units for moments (N m) in addition to the guidance given in section 2.2.6 of AS Unit 2. Learners should know that the coefficient of friction, μ , is dimensionless.
2.4.8 Kinematics	
Extend, use and derive the formulae for constant acceleration for motion in a straight line to 2 dimensions using vectors.	Learners should be able to recall and use the following formulae: <ul style="list-style-type: none"> • $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ • $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ (+\mathbf{r}_0, initial position) • $\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$ (+\mathbf{r}_0, initial position) Derivation of these formulae may require: <ul style="list-style-type: none"> • Substitution; • Use of calculus (see below).
Extend the use of calculus in kinematics for motion in a straight line to 2 dimensions using vectors.	To include the use of $\mathbf{v} = \frac{d\mathbf{r}}{dt}, \quad \mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2}, \quad \mathbf{r} = \int \mathbf{v} dt, \quad \mathbf{v} = \int \mathbf{a} dt$, where \mathbf{v} , \mathbf{a} and \mathbf{r} are given in terms of t . The required calculus will be consistent with that covered in sections 2.3.6 and 2.3.7 of A2 Unit 3.

Topics	Guidance
<p>Model motion under gravity in a vertical plane using vectors; projectiles.</p>	<p>Use of vector forms of the formulae for constant acceleration where $\mathbf{a} = -g\mathbf{j}$ and \mathbf{j} is perpendicular to the surface of the earth.</p> <p>The following results will relate to a particle being projected from the origin, with an initial velocity $V \text{ m s}^{-1}$, at an angle θ above the horizontal. In examination questions, learners may be expected to derive</p> <ul style="list-style-type: none"> • the general form of the formulae for the range, $R = \frac{V^2}{g} \sin 2\theta$; • the time of flight, $T = \frac{2V}{g} \sin \theta$; • the greatest height, $H = \frac{V^2}{2g} \sin^2 \theta$; • the equation of path (trajectory), $y = x \tan \theta - gx^2 \frac{(1 + \tan^2 \theta)}{2V^2}.$ <p>In questions where derivation of formulae has not been requested, the quoting of these formulae will not gain full credit.</p> <p>To include finding the speed and direction of motion of the projectile at any point on its path. The maximum horizontal range of a projectile for a given speed of projection.</p> <p>Questions will not involve resistive forces.</p>

Topics	Guidance
2.4.9 Forces and Newton's laws	
Extend Newton's second law to situations where forces need to be resolved (restricted to two dimensions).	Motion of a particle moving on an inclined plane.
Resolve forces in two dimensions. Understand and use the equilibrium of a particle under coplanar forces.	<p>Learners should be able to resolve forces</p> <ul style="list-style-type: none"> • horizontally and vertically; • parallel and perpendicular to an inclined plane; 
Understand and use addition of forces; resultant forces; dynamics for motion in a plane.	Several forces, given in magnitude-direction form, may need to be resolved into 2 components

Topics	Guidance
<p>Understand and use the $F \leq \mu R$ model for friction.</p> <p>The coefficient of friction.</p> <p>The motion of a body on a rough surface.</p> <p>Limiting friction and statics.</p>	<p>Forces will be constant and will include weight, friction, normal reaction, tension and thrust.</p> <p>To include motion on an inclined plane.</p> <p>Learners should be able to understand and use the concept of a normal reaction, R.</p> <p>Learners should be able to understand and use the fact that</p> <ul style="list-style-type: none"> • the direction of the frictional force will oppose the direction in which the body will move; • $F = \mu R$ when the body is moving; • $F \leq \mu R$ when the body is in equilibrium. <p>The motion of particles connected by strings passing over smooth, fixed pulleys or pegs; one particle will be freely hanging and the other particle may be on an inclined plane.</p> 

Topics	Guidance
2.4.10 Moments	
Understand and use moments in simple static contexts.	<p>To include parallel forces only.</p> <p>Learners should understand that a rigid body is in equilibrium when the resultant force is zero and the sum of the moments about any point will be zero.</p> <p>Use of both uniform and non-uniform rods.</p>
2.4.11 Vectors	
Understand and use vectors in three dimensions.	<p>To include the use of the unit vectors \mathbf{i}, \mathbf{j} and \mathbf{k}. Candidates may use column vectors for vectors in 2 or 3 dimensions.</p>
Use vectors to solve problems in context, including forces and kinematics.	<p>Questions will not involve the scalar product.</p>

Notes on modelling

Learners should be able to

- state any necessary modelling assumptions. For example,
 - objects can be modelled as particles (including projectiles),
 - strings will be light and inextensible,
 - 'rough' implies that friction must be considered,
 - no external forces are present unless otherwise stated, eg. air resistance can be ignored,
 - acceleration due to gravity, g , will be assumed to be constant, irrespective of height and vertical distance travelled.
- apply the equations of motion for the individual particles within a given system;
- identify possible limitations of any modelling assumptions made;
- briefly describe the mathematical consequences if certain modelling requirements are not met,
- suggest possible refinements to a chosen mathematical model.

Attempts should be made to relate questions to real-life settings wherever possible. For example, sporting situations for projectiles.

ASSESSMENT OBJECTIVES
ASSESSMENT OBJECTIVE 1 (AO1)

ASSESSMENT OBJECTIVES AO1: Use and apply standard techniques	Weighting	
	AS	A Level
Learners should be able to: <ul style="list-style-type: none"> select and correctly carry out routine procedures; and accurately recall facts, terminology and definitions 	45% – 55%	45% – 55%

Example of a question that is assessing AO1

SAMs Unit 1, Pure Mathematics

1. The circle C has centre A and equation

$$x^2 + y^2 - 2x + 6y - 15 = 0.$$

- (a) Find the coordinates of A and the radius of C . [3]
- (b) The point P has coordinates $(4, -7)$ and lies on C . Find the equation of the tangent to C at P . [4]

ASSESSMENT OBJECTIVE 2 (AO2)

ASSESSMENT OBJECTIVES AO2: Reason, interpret and communicate mathematically	Weighting	
	AS	A Level
<p>Learners should be able to:</p> <ul style="list-style-type: none"> construct rigorous mathematical arguments (including proofs); make deductions and inference; assess the validity of mathematical arguments; explain their reasoning; and use mathematical language and notation correctly. 	20% – 30%	20% – 30%

Example of a question that is assessing AO2

SAMs Unit 1, Pure Mathematics

6. In each of the two statements below, c and d are real numbers. One of the statements is true while the other is false.

A Given that $(2c + 1)^2 = (2d + 1)^2$, then $c = d$.

B Given that $(2c + 1)^3 = (2d + 1)^3$, then $c = d$.

- (a) Identify the statement which is false. Find a counter example to show that this statement is in fact false.
- (b) Identify the statement which is true. Give a proof to show that this statement is in fact true. [5]

ASSESSMENT OBJECTIVE 3 (AO3)

ASSESSMENT OBJECTIVES AO3: Solve problems within mathematics and in other contexts	Weighting	
	AS	A Level
<p>Learners should be able to:</p> <ul style="list-style-type: none"> translate problems in mathematical and non-mathematical contexts into mathematical processes; interpret solutions to problems in their original context, and, where appropriate, evaluate their accuracy and limitations; translate situations in context into mathematical models; use mathematical models; and evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them. 	20% – 30%	20% – 30%

Mathematical Problem Solving

Attributes of a problem solving task:

- Little or no scaffolding.
- Multiple representations, eg. sketch/diagram as well as calculations.
- Information is not given in mathematical form/language; or interpretation of results or evaluation of methods in a real world context.
- Variety of techniques that could be used.
- Solution requires understanding of the processes involved.
- Two or more mathematical processes required or different parts of mathematics to be brought together to reach a solution.

(A Level Mathematics Working Group Report, December 2015, Ofqual/15/5789)

Examples of questions that are assessing AO3

SAMs Unit 3, Pure Mathematics

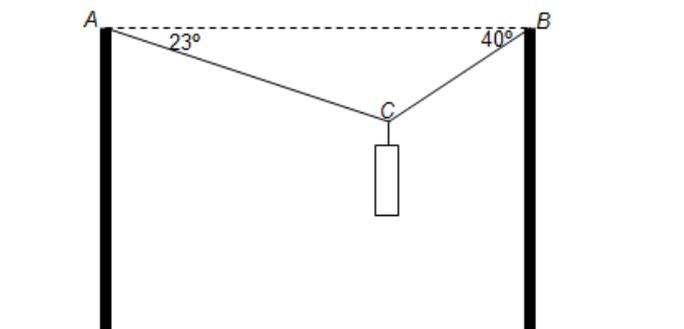
2. Air is pumped into a spherical balloon at the rate of 250 cm^3 per second. When the radius of the balloon is 15 cm , calculate the rate at which the radius is increasing, giving your answer to three decimal places [3]

8. (b) Use an appropriate substitution to show that

$$\int_0^{\frac{1}{2}} \frac{x^2}{\sqrt{1-x^2}} dx = \frac{\pi}{12} - \frac{\sqrt{3}}{8}. \quad [8]$$

SAMs Unit 4, Applied Mathematics, Section B

7. The diagram below shows an object of weight 160 N at a point C , supported by two cables AC and BC inclined at angles of 23° and 40° to the horizontal respectively.



- (a) Find the tension in AC and the tension in BC . [6]
- (b) State two modelling assumptions you have made in your solution. [2]

NOTES ON NEW TOPICS
WJEC GCE A LEVEL IN MATHEMATICS

AS Unit 1: Pure Mathematics A

- Trigonometry (i) Prove trigonometric identities.
- Vectors (i) Vectors in 2-D.

A2 Unit 3: Pure Mathematics B

- Algebra and Functions (i) Partial fractions with three linear terms in the denominator.
- Trigonometry (i) Small angle approximations.
(ii) Prove trigonometric identities.
- Differentiation (i) Differentiate $\sin x$ and $\cos x$ from first principles.
(ii) Differentiate a^{kx}
(iii) Connected rates of change.
- Numerical Methods (i) Iterative method $x = g(x)$, cobweb and staircase diagrams.
(ii) Newton-Raphson method.

AS Unit 1: Trigonometry: Prove trigonometric identities.

Example

1. a) Show that $5\cos^2\theta - 3\sin^2\theta \equiv 8\cos^2\theta - 3$.
- b) Hence solve the equation $5\cos^2\theta - 3\sin^2\theta = -1$ for values of θ between 0° and 180° .

Solution

$$\begin{aligned} \text{a) } 5\cos^2\theta - 3\sin^2\theta &\equiv 5\cos^2\theta - 3(1 - \cos^2\theta) \\ &= 5\cos^2\theta - 3 + 3\cos^2\theta \\ &= 8\cos^2\theta - 3. \end{aligned}$$

$$\begin{aligned} \text{b) } 8\cos^2\theta - 3 &= -1 \\ 8\cos^2\theta &= 2 \end{aligned}$$

$$\cos^2\theta = \frac{1}{4} \rightarrow \cos\theta = \pm\frac{1}{2}$$

$$\theta = 60^\circ, 120^\circ.$$

Example

1. A, B, C are three points given by position vectors $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$, $\mathbf{b} = 14\mathbf{i} - 2\mathbf{j}$, $\mathbf{c} = -8\mathbf{i} + 3\mathbf{j}$ with respect to the origin O.
- Find \mathbf{AB} in terms of \mathbf{i} and \mathbf{j} .
 - Find AB , the magnitude of \mathbf{AB} .
 - Find the position vector of M , the midpoint of BC .
 - Find the position vector of the point P which divides AC in the ratio 3 : 7.

Solution

a) $\mathbf{AB} = \mathbf{b} - \mathbf{a} = 14\mathbf{i} - 2\mathbf{j} - (2\mathbf{i} + 3\mathbf{j}) = 12\mathbf{i} - 5\mathbf{j}$.

b) $|\mathbf{AB}| = \sqrt{12^2 + (-5)^2} = 13$

- c) M is the midpoint of BC , therefore

$$\mathbf{BM} = \mathbf{MC}$$

$$\mathbf{m} - \mathbf{b} = \mathbf{c} - \mathbf{m}$$

$$2\mathbf{m} = \mathbf{c} + \mathbf{b} \text{ so that } \mathbf{m} = \frac{1}{2}(\mathbf{c} + \mathbf{b})$$

$$\mathbf{m} = \frac{1}{2}(-8\mathbf{i} + 3\mathbf{j} + 14\mathbf{i} - 2\mathbf{j}) = 3\mathbf{i} + \frac{1}{2}\mathbf{j}$$

- d) P divides AC in the ratio 3 : 7 therefore $AP : PC = 3 : 7$

$$7\mathbf{AP} = 3\mathbf{PC}$$

$$7(\mathbf{p} - \mathbf{a}) = 3(\mathbf{c} - \mathbf{p})$$

$$10\mathbf{p} = 3\mathbf{c} + 7\mathbf{a} = 3(-8\mathbf{i} + 3\mathbf{j}) + 7(2\mathbf{i} + 3\mathbf{j})$$

$$\mathbf{p} = -\mathbf{i} + 3\mathbf{j}$$

A2 Unit 3: Algebra and Functions: Partial fractions

Example

1. Express $\frac{x+1}{(x-1)(x-2)(x-3)}$ in partial fractions.

Solution

$$\frac{x+1}{(x-1)(x-2)(x-3)} \equiv \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$x+1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

$$x=1: 2 = A(-1)(-2) \quad A = 1$$

$$x=2: 3 = B(1)(-1) \quad B = -3$$

$$x=3: 4 = C(2)(1) \quad C = 2$$

$$\frac{x+1}{(x-1)(x-2)(x-3)} = \frac{1}{x-1} - \frac{3}{x-2} + \frac{2}{x-3}$$

A2 Unit 3: Trigonometry: Small angle approximations

Example

1. Use the approximations $\sin x \approx x$, $\tan x \approx x$, $\cos x \approx 1 - \frac{1}{2}x^2$ to find the smallest positive root of the equation $\sin x + \cos x + \tan x = 1.5$.
Give your answer correct to 3 decimal places.

Solution

$$x + 1 - \frac{1}{2}x^2 + x = 1.5$$

$$\frac{1}{2}x^2 - 2x + 0.5 = 0$$

$$x^2 - 4x + 1 = 0$$

$$x = \frac{4 \pm \sqrt{12}}{2} = 3.732 \text{ or } 0.268$$

therefore $x = 0.268$ to 3 decimal places.

A2 Unit 3: Trigonometry: Prove trigonometric identities

Example

1. Show that $\tan A + \cot A \equiv 2 \operatorname{cosec} 2A$

Solution

$$\begin{aligned} \tan A + \cot A &= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \\ &= \frac{\sin^2 A + \cos^2 A}{\cos A \sin A} \\ &= \frac{1}{\sin A \cos A} = \frac{2}{2 \sin A \cos A} \\ &= \frac{2}{\sin 2A} = 2 \operatorname{cosec} 2A \end{aligned}$$

A2 Unit 3: Differentiation

Examples

1. Differentiate $\sin x$ from first principles.

Solution

$$\text{Let } f(x) = \sin x$$

$$f(x+h) = \sin(x+h) = \sin x \cosh + \sinh \cos x$$

$$\text{when } h \text{ is very small } \sinh \approx h \text{ and } \cosh \approx 1 - \frac{1}{2}h^2$$

$$\text{so that } f(x+h) = \sin x(1 - \frac{1}{2}h^2) + h \cos x = \sin x - \frac{1}{2}h^2 \sin x + h \cos x$$

$$f(x+h) - f(x) = -\frac{1}{2}h^2 \sin x + h \cos x$$

$$\frac{f(x+h) - f(x)}{h} = -\frac{1}{2}h \sin x + \cos x$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = f'(x) = \cos x$$

2. Differentiate 5^x with respect to x .

Solution

$$\text{Let } f(x) = 5^x$$

$$\text{Taking logs: } \ln f(x) = \ln 5^x$$

$$\ln f(x) = x \ln 5$$

$$\text{differentiating } \frac{f'(x)}{f(x)} = \ln 5$$

$$f'(x) = f(x) \ln 5 = 5^x \ln 5$$

3. The area, $A \text{ cm}^2$, of a circular blot of ink is increasing at a rate of $3 \text{ cm}^2/\text{sec}$. Find the rate at which the radius is increasing when the radius is 2 cm , giving your answer correct to 2 decimal places.

Solution

Since $A = \pi r^2$

$$\frac{dA}{dr} = 2\pi r$$

$$\frac{dr}{dt} = \frac{dr}{dA} \times \frac{dA}{dt} = \frac{1}{2\pi r} \times 3 = \frac{3}{2\pi r}$$

When $r = 2$

$$\frac{dr}{dt} = \frac{3}{4\pi} = 0.24 \text{ cm/sec to 2 decimal places.}$$

A2 Unit 3: Numerical Methods

Examples

1. The equation $e^x + x - 3 = 0$ has a positive root α .
 - a) Show that α lies between 0.5 and 1.
 - b) Use the iterative formula $x_{n+1} = \ln(3 - x_n)$ and $x_0 = 1$ to find α correct to 3 decimal places.
 - c) Use the Newton-Raphson method to find α correct to 5 decimal places.

Solution

a) Let $f(x) = e^x + x - 3$

$$f(0.5) = -0.851278... < 0$$

$$f(1) = 0.718281... > 0$$

change of sign, therefore the root α is between 0.5 and 1.

b) Using $x_{n+1} = \ln(3 - x_n)$ and $x_0 = 1$

$$x_1 = 0.6931471806 ..$$

$$x_2 = 0.8358841798 ..$$

$$x_3 = 0.7720118809 ..$$

$$x_4 = 0.8010989895 ..$$

$$x_5 = 0.7879576949 ..$$

$$x_6 = 0.7939162088 ..$$

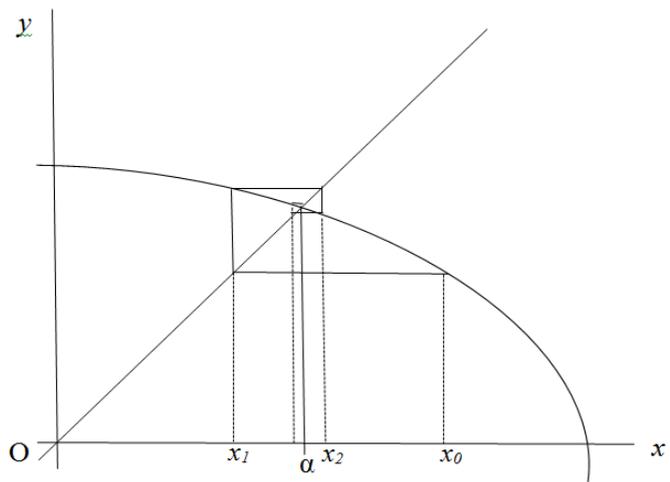
$$x_7 = 0.7912189034 ..$$

$$x_8 = 0.7924408234 ..$$

$$x_9 = 0.7918874602..$$

therefore $\alpha = 0.792$ to 3 decimal places.

This cobweb diagram illustrates how the values converge towards the root α .



c) Using Newton-Raphson:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{e^{x_n} + x_n - 3}{e^{x_n} + 1}$$

$$x_0 = 1$$

$$x_1 = 0.8068242641 \dots$$

$$x_2 = 0.7921349597 \dots$$

$$x_3 = 0.7920599704 \dots$$

$$x_4 = 0.7920599684 \dots$$

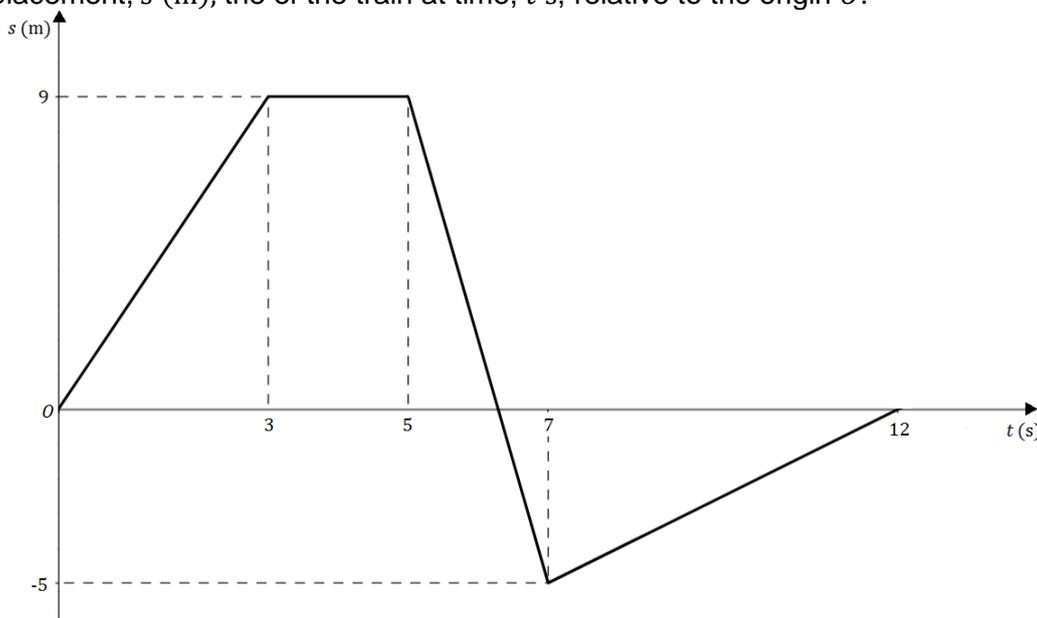
therefore $\alpha = 0.79206$ to 5 decimal places.

Additional Examples

AS Unit 2: Kinematics: graphs for motion in a straight line

Example

1. The diagram below shows a sketch of the displacement-time graph of a train running along a straight horizontal track for 12 s. The vertical axis represents the displacement, s (m), the of the train at time, t s, relative to the origin O .



- (a) Find the greatest speed obtained by the train.
- (b) Determine the total distance travelled and hence find the average speed of the train.

Solution

- (a) Use velocity = gradient of graph.
The magnitude of the gradient is greatest for $5 < t < 7$.
Therefore, for $5 < t < 7$,
velocity = $\frac{-9-5}{7-5} = \frac{-14}{2} = -7$ \therefore greatest speed = 7 m s^{-1} .
- (b) Distance travelled = $9 + 9 + 5 + 5 = 28 \text{ m}$.
Average speed = $\frac{\text{total distance travelled}}{\text{total time taken}} = \frac{28}{12} = 2.33 \text{ m s}^{-1}$ (2 dp)

2. Geraint is a sprinter. At $t = 0$ seconds he is stationary at the starting line. His coach decides to model his acceleration, $a \text{ m s}^{-2}$, by using

$$a = \begin{cases} 6 - 2t & \text{for } 0 \leq t \leq 3 \\ 0 & \text{for } t > 3 \end{cases}$$

- (a) Show that Geraint's velocity, m s^{-1} , is given by

$$v = \begin{cases} 6t - t^2 & \text{for } 0 \leq t \leq 3 \\ 9 & \text{for } t > 3 \end{cases}$$

- (b) Sketch a velocity time graph for a race lasting 20 s.
 (c) Calculate the time that Geraint takes to run 100 m.

Solution

- (a) For $0 \leq t \leq 3$,

$$v = \int a \, dt = \int (6 - 2t) \, dt$$

$$v = 6t - t^2 + c$$

At $t = 0$, $v = 0$ so that $c = 0$.

$$\therefore v = 6t - t^2$$

When $t = 3$, $v = 9$ and $a = 0$ for $t > 3$ so that $v = 9$ for $t > 3$.

- (b) For $0 \leq t \leq 3$, by completing the square, we get

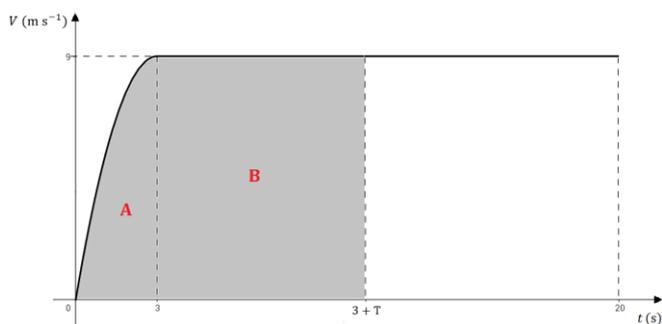
$$v = 6t - t^2 = 9 - (t - 3)^2$$

Therefore, v attains a maximum value of 9 at $t = 3$. Also, $v = 0$ at $t = 0$.



- (c) We need a value of T such that

distance travelled = area under graph = shaded area A + shaded area B = 100 m



$$\text{Area A} = \int_0^3 (6t - t^2) dt = \left[3t^2 - \frac{t^3}{3} \right]_0^3 = 18$$

$$\therefore 9T + 18 = 100$$

$$T = \frac{82}{9}$$

$$\text{Area B} = 9T$$

Therefore, time taken to run 100 m is

$$3 + \frac{82}{9} = \frac{109}{9} = 12.11 \text{ s (2 dp).}$$

A2 Unit 4: Kinematics: Projectiles using Vectors in 2-D

Example

1. A particle is projected from the origin O with initial velocity \mathbf{u} m s⁻¹. After 4 seconds, the particle has position vector $(60\mathbf{i} + 25.6\mathbf{j})$ m.
- Calculate the distance of the particle from the origin after 4 seconds.
 - Find the value of \mathbf{u} , giving your answer in the form $x\mathbf{i} + y\mathbf{j}$.
 - Hence determine
 - the minimum speed of the particle,
 - the speed of projection of the particle together with the angle it makes with the horizontal.

Solution

- (a) At $t = 4$, $\mathbf{s} = 60\mathbf{i} + 25.6\mathbf{j}$,
 distance of the particle from $O = |\mathbf{s}| = \sqrt{(60)^2 + (25.6)^2} = 65.23$ m
- (b) Use $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with $\mathbf{s} = 60\mathbf{i} + 25.6\mathbf{j}$, $\mathbf{a} = -g\mathbf{j}$ and $t = 4$.
 $60\mathbf{i} + 25.6\mathbf{j} = \mathbf{u}(4) + \frac{1}{2}(-g\mathbf{j})(4)^2$
 $60\mathbf{i} + 25.6\mathbf{j} = 4\mathbf{u} - 8g\mathbf{j}$
 $\mathbf{u} = (15\mathbf{i} + 26\mathbf{j})$ m s⁻¹
- (c) (i) At the minimum speed, the vertical component of velocity (i.e. the \mathbf{j} coefficient) will be zero, therefore
 minimum speed = 15 m s⁻¹
- (c) (ii) speed of projection of the particle = $|\mathbf{u}| = \sqrt{(15)^2 + (26)^2} = 30.02$ m s⁻¹

$$\tan \theta = \frac{26}{15} \Rightarrow \theta = 60.02^\circ$$

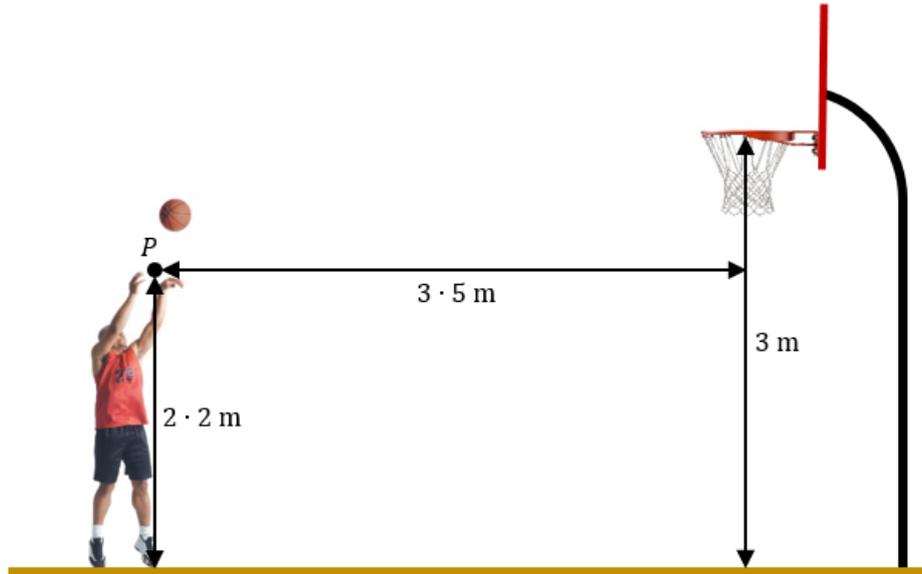
Direction is 60.02° **above** the horizontal.

Notation: Alternatively, \mathbf{r} may be used to represent the position vector.

Example

2. David is playing basketball. He throws the ball with velocity $(7\mathbf{i} + 4 \cdot 05\mathbf{j}) \text{ m s}^{-1}$ from a point P which is at a height of $2 \cdot 2 \text{ m}$ above the horizontal. The centre of the hoop is 3 m above ground level and a horizontal distance of $3 \cdot 5 \text{ m}$ from the point of projection.

The unit vectors \mathbf{i} and \mathbf{j} are horizontal and vertical respectively. You may also assume that there are no external forces acting on the ball.



- (a) By considering the point Q with position vector $(3 \cdot 5\mathbf{i} + x\mathbf{j}) \text{ m}$,
- find the time taken for the particle to travel from P to Q ,
 - show that the value of $x = 0 \cdot 8 \text{ m}$,
 - calculate the speed and direction of the ball as it passes through Q and use it to determine if the ball will pass through the hoop, giving a reason for your answer.
- (b) In addition to the assumption given in the question, write down one further assumption that you have made in your solution to this problem and explain how this assumption affects your conclusion in (a) (iii).

Solution

- (a) (i) Using $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with $\mathbf{s} = 3 \cdot 5\mathbf{i} + x\mathbf{j}$, $\mathbf{u} = 7\mathbf{i} + 4 \cdot 05\mathbf{j}$, and $\mathbf{a} = -g\mathbf{j}$,

$$3 \cdot 5\mathbf{i} + x\mathbf{j} = (7\mathbf{i} + 4 \cdot 05\mathbf{j})t + \frac{1}{2}(-g\mathbf{j})t^2$$

Comparison of \mathbf{i} coefficients yields

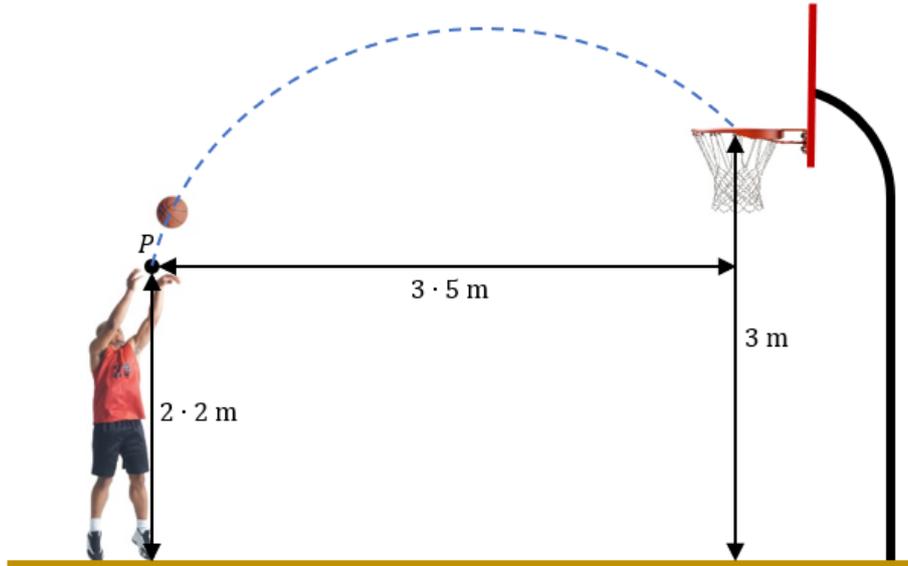
$$7t = 3 \cdot 5 \text{ so that } t = \frac{3 \cdot 5}{7} = 0 \cdot 5 \text{ s}$$

- (ii) Comparison of \mathbf{j} coefficients yields

$$x = 4 \cdot 05t - \frac{gt^2}{2}$$

$$\text{At } t = 0 \cdot 5, x = 4 \cdot 05(0 \cdot 5) - \frac{g(0 \cdot 5)^2}{2} = 0 \cdot 8 \text{ m.}$$

(iii)



Using $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ with $\mathbf{u} = 7\mathbf{i} + 4.05\mathbf{j}$, $\mathbf{a} = -g\mathbf{j}$, and $t = 0.5$

$$\mathbf{v} = (7\mathbf{i} + 4.05\mathbf{j}) + (-g\mathbf{j})(0.5) = 7\mathbf{i} - 0.85\mathbf{j}$$

speed of the particle at Q = $|\mathbf{v}| = \sqrt{(7)^2 + (-0.85)^2} = 7.05 \text{ m s}^{-1}$

$$\tan \theta = \frac{-0.85}{7} \Rightarrow \theta = -6.92^\circ$$

Direction is 6.92° **below** the horizontal.

Since the ball is on the downwards part of its trajectory and it travels through $(3.5\mathbf{i} + 0.8\mathbf{j})$, which represents the position vector for the centre of the hoop relative to P, then it will pass through the hoop.

- (b) It is assumed that the ball is a particle. However, the dimensions of a basketball are significant. Therefore, due to the shallow angle that the ball approaches the hoop, it is likely that contact will be made with the hoop and so it may not actually pass through it.