GCE AS/A LEVEL

WJEC GCE AS/A Level in
MATHEMATICS

APPROVED BY QUALIFICATIONS WALES

SAMPLE ASSESSMENT MATERIALS

Teaching from 2017

For teaching from 2017
For award from 2018

GCE AS AND A LEVEL MATHEMATICS

SAMPLE ASSESSMENT MATERIALS

Contents

GCE

MATHEMATICS

UNIT 1: PURE MATHEMATICS A

SAMPLE ASSESSMENT MATERIALS

(2 hour 30 minutes)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

1. The circle *C* has centre *A* and equation

$$x^2 + y^2 - 2x + 6y - 15 = 0.$$

(*a*) Find the coordinates of *A* and the radius of *C*. [3]

(*b*) The point *P* has coordinates (4, – 7) and lies on *C*. Find the equation of the tangent to *C* at *P*. [4]

2. Find all values of θ between 0° and 360° satisfying

$$7 \sin^2\theta + 1 = 3 \cos^2\theta - \sin \theta. \quad [6]$$

3. Given that $y = x^3$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ from first principles. [6]

4. The cubic polynomial $f(x)$ is given by $f(x) = 2x^3 + ax^2 + bx + c$, where a, b, c are constants. The graph of $f(x)$ intersects the x-axis at the points with coordinates (– 3, 0), (2·5, 0) and (4, 0). Find the coordinates of the point where the graph of $f(x)$ intersects the y-axis. [5]

5. The points *A*(0, 2), *B*(– 2, 8), *C*(20, 12) are the vertices of the triangle *ABC*.
The point *D* is the mid-point of *AB*.

(*a*) Show that *CD* is perpendicular to *AB*. [6]

(*b*) Find the exact value of tan $C\hat{A}B$. [5]

(*c*) Write down the geometrical name for the triangle *ABC*. [1]

6. In each of the two statements below, c and d are real numbers. One of the statements is true while the other is false.

A Given that $(2c + 1)^2 = (2d + 1)^2$, then $c = d$.
B Given that $(2c + 1)^3 = (2d + 1)^3$, then $c = d$.

(*a*) Identify the statement which is false. Find a counter example to show that this statement is in fact false.

(*b*) Identify the statement which is true. Give a proof to show that this statement is in fact true. [5]

7. Figure 1 shows a sketch of the graph of $y = f(x)$. The graph has a minimum point at $(-3, -4)$ and intersects the x-axis at the points $(-8, 0)$ and $(2, 0)$.

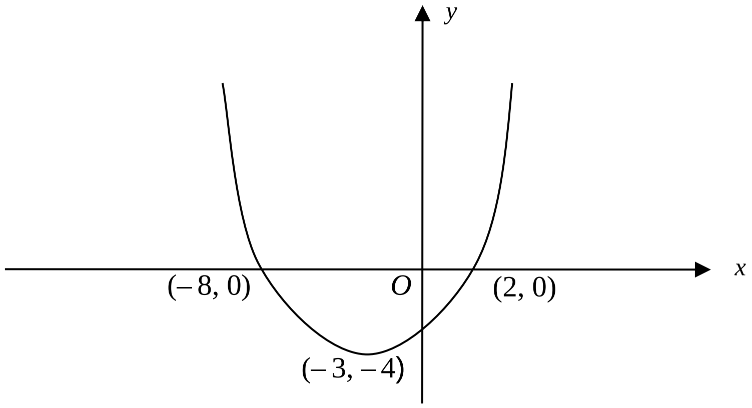


Figure 1

(*a*) Sketch the graph of $y = f(x + 3)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. [3]

(*b*) Figure 2 shows a sketch of the graph having **one** of the following equations with an appropriate value of either p, q or r.

$y = f(px)$, where p is a constant
$y = f(x) + q$, where q is a constant
$y = rf(x)$, where r is a constant

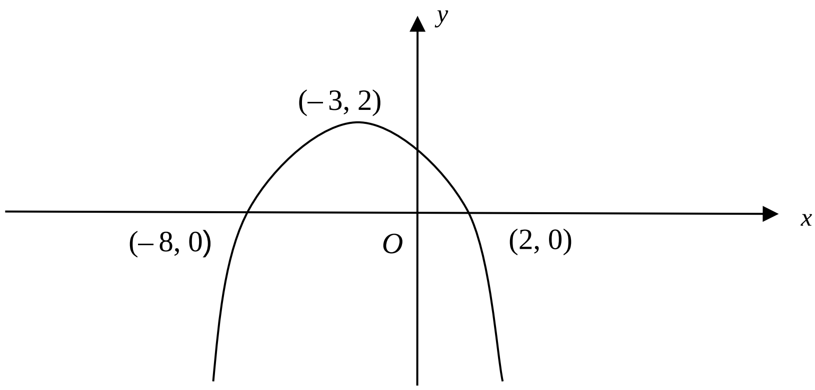


Figure 2

Write down the equation of the graph sketched in Figure 2, together with the value of the corresponding constant. [2]

8. The circle *C* has radius 5 and its centre is the origin.
The point *T* has coordinates (11, 0).
The tangents from *T* to the circle *C* touch *C* at the points *R* and *S*.

(*a*) Write down the geometrical name for the quadrilateral *ORTS*. [1]

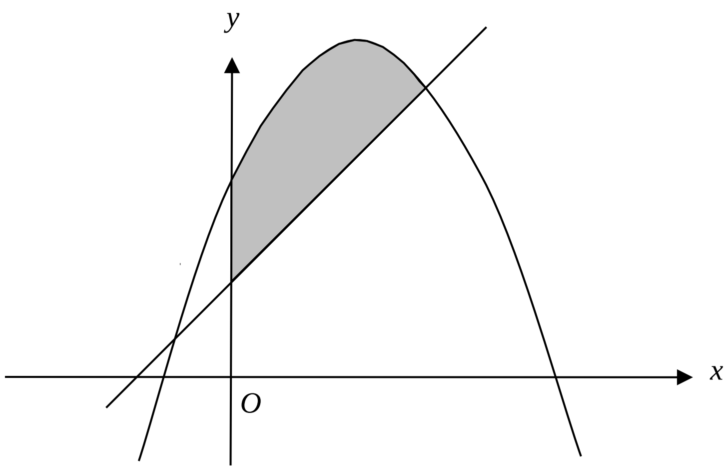
(*b*) Find the exact value of the area of the quadrilateral *ORTS*. Give your answer in its simplest form. [5]

9. The quadratic equation $4x^2 - 12x + m = 0$, where m is a positive constant, has **two distinct** real roots.
Show that the quadratic equation $3x^2 + mx + 7 = 0$ has **no** real roots. [7]

10. (*a*) **Use the binomial theorem** to express $\left(\sqrt{3} - \sqrt{2}\right)^5$ in the form $a\sqrt{3} + b\sqrt{2}$, where a, b are integers whose values are to be found. [5]

(*b*) Given that $\left(\sqrt{3} - \sqrt{2}\right)^5 \approx 0$, use your answer to part (*a*) to find an approximate value for $\sqrt{6}$ in the form $\frac{c}{d}$, where c and d are positive integers whose values are to be found. [3]

11.



The diagram shows a sketch of the curve $y = 6 + 4x - x^2$ and the line $y = x + 2$. The point *P* has coordinates (a, b). Write down the three inequalities involving a and b which are such that the point *P* will be strictly contained within the shaded area above, if and only if, all three inequalities are satisfied. [3]

12. Prove that

$$\log_7 a \times \log_a 19 = \log_7 19$$

whatever the value of the positive constant a. [3]

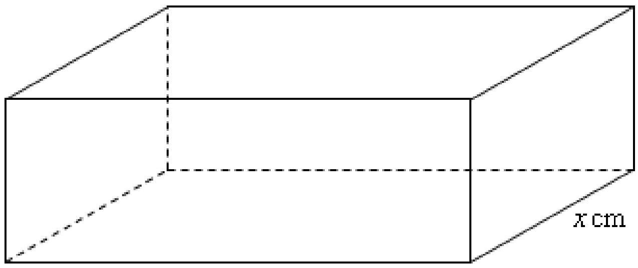
13. In triangle ABC, BC = 12 cm and $\cos A\hat{B}C = \frac{2}{3}$.

The length of AC is 2 cm greater than the length of AB.

(*a*) Find the lengths of AB and AC. [4]

(*b*) Find the exact value of $\sin B\hat{A}C$. Give your answer in its simplest form. [3]

14. The diagram below shows a closed box in the form of a cuboid, which is such that the length of its base is twice the width of its base. The volume of the box is 9000 cm^3. The total surface area of the box is denoted by $S\,\text{cm}^2$.



(*a*) Show that $S = 4x^2 + \frac{27000}{x}$, where x cm denotes the width of the base. [3]

(*b*) Find the minimum value of S, showing that the value you have found is a minimum value. [5]

15. The size N of the population of a small island at time t years may be modelled by $N = Ae^{kt}$, where A and k are constants. It is known that N = 100 when t = 2 and that N = 160 when t = 12.

(*a*) Interpret the constant A in the context of the question. [1]

(*b*) Show that k = 0·047, correct to three decimal places. [4]

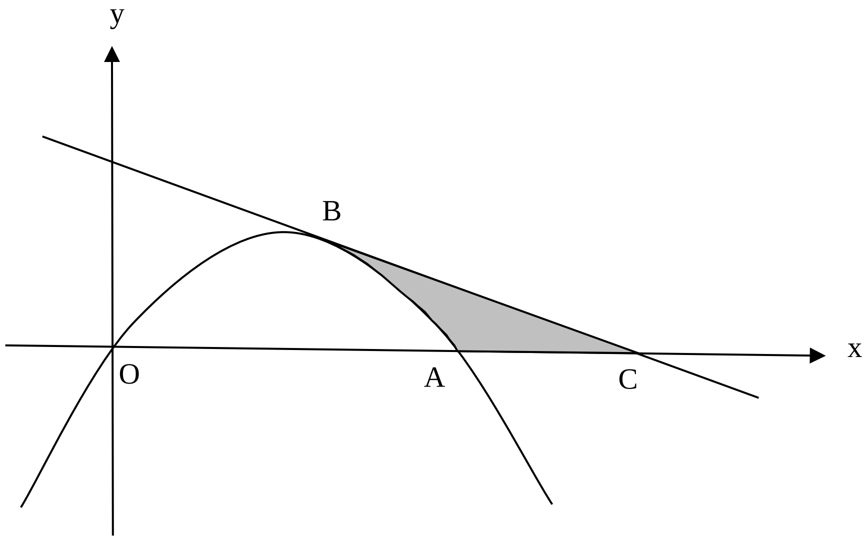
(*c*) Find the size of the population when t = 20. [3]

16. Find the range of values of x for which the function

$$f(x) = x^3 - 5x^2 - 8x + 13$$

is an increasing function. [5]

17.



The diagram above shows a sketch of the curve $y = 3x - x^2$. The curve intersects the x-axis at the origin and at the point A. The tangent to the curve at the point $B(2, 2)$ intersects the x-axis at the point C.

(*a*) Find the equation of the tangent to the curve at B. [4]

(*b*) Find the area of the shaded region. [8]

18. (*a*) The vectors $\mathbf{u}$ and $\mathbf{v}$ are defined by $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j}$, $\mathbf{v} = -4\mathbf{i} + 5\mathbf{j}$.

(i) Find the vector $4\mathbf{u} - 3\mathbf{v}$.

(ii) The vectors $\mathbf{u}$ and $\mathbf{v}$ are the position vectors of the points U and V, respectively. Find the length of the line UV. [4]

(*b*) Two villages A and B are 40 km apart on a long straight road passing through a desert. The position vectors of A and B are denoted by $\mathbf{a}$ and $\mathbf{b}$, respectively.

(i) Village C lies on the road between A and B at a distance 4 km from B. Find the position vector of C in terms of $\mathbf{a}$ and $\mathbf{b}$.

(ii) Village D has position vector $\frac{2}{9}\mathbf{a} + \frac{5}{9}\mathbf{b}$. Explain why village D cannot possibly be on the straight road passing through A and B. [3]

AS Mathematics Unit 1: Pure Mathematics A

General instructions for marking GCE Mathematics

1. The mark scheme should be applied precisely and no departure made from it. Marks should be awarded directly as indicated and no further subdivision made.

2. Marking Abbreviations
The following may be used in marking schemes or in the marking of scripts to indicate reasons for the marks awarded.

cao = correct answer only
MR = misread
PA = premature approximation
bod = benefit of doubt
oe = or equivalent
si = seen or implied
ISW = ignore subsequent working

F.T. = follow through (✓ indicates correct working following an error and ✗ indicates a further error has been made)

Anything given in brackets in the marking scheme is expected but, not required, to gain credit.

3. Premature Approximation
A candidate who approximates prematurely and then proceeds correctly to a final answer loses 1 mark as directed by the Principal Examiner.

4. Misreads
When the data of a question is misread in such a way as not to alter the aim or difficulty of a question, follow through the working and allot marks for the candidates' answers as on the scheme using the new data.
This is only applicable if a wrong value, is used consistently throughout a solution; if the correct value appears anywhere, the solution is not classed as MR (but may, of course, still earn other marks).

5. Marking codes

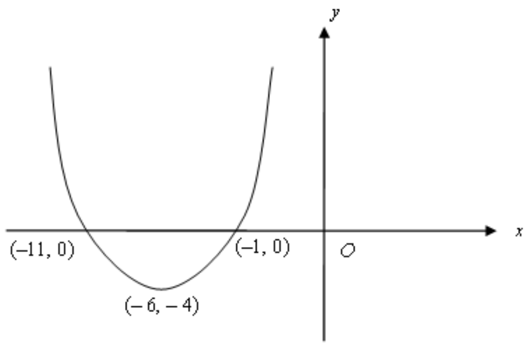
- ‘M' marks are awarded for any correct method applied to appropriate working, even though a numerical error may be involved. Once earned they cannot be lost.
- ‘m’ marks are dependant method marks. They are only given if the relevant previous ‘M’ mark has been earned.
- ‘A' marks are given for a numerically correct stage, for a correct result or for an answer lying within a specified range. They are only given if the relevant M/m mark has been earned either explicitly or by inference from the correct answer.
- 'B' marks are independent of method and are usually awarded for an accurate result or statement.
- ‘S’ marks are awarded for strategy
- ‘E’ marks are awarded for explanation
- ‘U’ marks are awarded for units
- ‘P’ marks are awarded for plotting points
- ‘C’ marks are awarded for drawing curves

AS Mathematics Unit 1: Pure Mathematics A

Solutions and Mark Scheme

Question Number	Solution	Mark	AO	Notes
1. (a)	$A(1, -3)$	B1	AO1	
	A correct method for finding the radius, e.g., trying to rewrite the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$	M1	AO1	
	Radius = 5	A1	AO1	
(b)	Gradient $AP = \dfrac{\text{increase in } y}{\text{increase in } x}$	M1	AO1	
	Gradient $AP = \dfrac{(-7)-(-3)}{4-1} = -\dfrac{4}{3}$	A1	AO1	(f.t. candidate's coordinates for A)
	Use of $m_{\text{tan}} \times m_{\text{rad}} = -1$	M1	AO1	
	Equation of tangent is: $y-(-7) = \dfrac{3}{4}(x-4)$	A1 **[7]**	AO1	(f.t. candidate's gradient for AP)
2.	$7\sin^2\theta + 1 = 3(1-\sin^2\theta) - \sin^2\theta$	M1	AO1	(correct use of $\cos^2\theta = 1-\sin^2\theta$)
	An attempt to collect terms, form and solve a quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c$ = candidate's coefficient of $\sin^2\theta$ and $b \times d$ = candidate's constant	m1	AO1	
	$10\sin^2\theta + \sin\theta - 2 = 0$ $\Rightarrow (2\sin\theta + 1)(5\sin\theta - 2) = 0$ $\Rightarrow \sin\theta = -\dfrac{1}{2}, \sin\theta = \dfrac{2}{5}$	A1	AO1	(c.a.o.)
	$\theta = 210°, 330°$	B1 B1	AO1 AO1	
	$\theta = 23{\cdot}57(8178\ldots)°, 156{\cdot}42(182\ldots)°$	B1	AO1	
	Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range. $\sin\theta = +, -$, f.t. for 3 marks, $\sin\theta = -, -$, f.t. for 2 marks $\sin\theta = +, +$, f.t. for 1 mark	**[6]**		

Question Number	Solution	Mark	AO	Notes
3.	$y + k = (x + h)^3$ $y + k = x^3 + 3x^2h + 3xh^2 + h^3$ Subtracting y from above to find k $k = 3x^2h + 3xh^2 + h^3$ Dividing by h and letting $h \to 0$ $\frac{dy}{dx} = \lim_{h \to 0} \frac{k}{h} = 3x^2$	M1 A1 M1 A1 M1 A1 **[6]**	AO2 AO2 AO2 AO2 AO2 AO2	 (c.a.o.)
4.	Correct use of the Factor Theorem to find at least one factor of $f(x)$ At least two factors of $f(x)$ $f(x) = (x + 3)(x - 4)(2x - 5)$ Use of the fact that $f(x)$ intersects the y-axis when $x = 0$ $f(x)$ intersects the y-axis at (0, 60)	M1 A1 A1 M1 A1 **[5]**	AO3 AO3 AO3 AO3 AO3	 (accept $(x - 2\cdot5)$ as a factor) (c.a.o.) (f.t. candidate's expression for $f(x)$)
5. (a)	A correct method for finding the coordinates of the mid-point of AB D has coordinates $(-1, 5)$ Gradient of $AB = \frac{\text{increase in } y}{\text{increase in } x}$ Gradient of $AB = -\frac{6}{2}$ Gradient of $CD = \frac{\text{increase in } y}{\text{increase in } x}$ Gradient of $CD = \frac{7}{21}$ $-\frac{6}{2} \times \frac{7}{21} = -1 \Rightarrow$ AB is perpendicular to CD	M1 A1 M1 A1 (M1) A1 B1	AO1 AO1 AO1 AO1 (AO1) AO1 AO2	 (or equivalent) (to be awarded only if the previous M1 is not awarded) (or equivalent)
(b)	A correct method for finding the length of AD or CD $AD = \sqrt{10}$ $CD = \sqrt{490}$ $\tan C\hat{A}B = \frac{CD}{AD}$ $\tan C\hat{A}B = 7$	M1 A1 A1 M1 A1	AO1 AO1 AO1 AO1 AO1	
(c)	Isosceles	B1 **[12]**	AO2	

Question Number	Solution	Mark	AO	Notes
6. (a)	For statement A Choice of $c \neq -{}^{1}/_{2}$ and $d = -c - 1$ Correct verification that given equation is satisfied	 M1 A1	 AO2 AO2	
(b)	For statement B Use of the fact that any real number has an unique real cube root $(2c + 1)^3 = (2d + 1)^3 \Rightarrow 2c + 1 = 2d + 1$ $2c + 1 = 2d + 1 \Rightarrow c = d$	 M1 A1 A1 **[5]**	 AO2 AO2 AO2	
7. (a)				
	Concave up curve and y-coordinate of minimum = – 4 x-coordinate of minimum = – 6 Both points of intersection with x -axis	 B1 B1 B1	 AO1 AO1 AO1	
(b)	$y = -\dfrac{1}{2}\, f(x)$ **If B2 not awarded** $y = rf(x)$ with r negative	B2 (B1) **[5]**	AO2 AO2 (AO2)	

Question Number		Solution	Mark	AO	Notes
8.	(a)	A kite	B1	AO2	
	(b)	A correct method for finding *TR*(*TS*)	M1	AO3	
		TR(*TS*) = $\sqrt{96}$	A1	AO3	
		Area *OTR*(*OTS*) = $\frac{1}{2}\times\sqrt{96}\times 5$	M1	AO3	(f.t. candidate's derived value for *TR*(*TS*))
		Area *OTRS* = 2 × Area *OTR*(*OTS*)	m1	AO3	
		Area *OTRS* = $20\sqrt{6}$	A1	AO3	(c.a.o.)
			[6]		
9.		An expression for $b^2 - 4ac$ for the quadratic equation $4x^2 - 12x + m = 0$, with at least two of a, b or c correct	M1	AO1	
		$b^2 - 4ac = 12^2 - 4 \times 4 \times m$	A1	AO1	
		$b^2 - 4ac > 0$	m1	AO1	
		$(0<)\ m < 9$	A1	AO1	
		An expression for $b^2 - 4ac$ for the quadratic equation $3x^2 + mx + 7 = 0$, with at least two of a, b or c correct	(M1)		(to be awarded only if the corresponding M1 is not awarded above)
		$b^2 - 4ac = m^2 - 84$	A1	AO2	
		$m^2 < 81 \Rightarrow b^2 - 4ac < -3$	A1	AO2	
		$b^2 - 4ac < 0 \Rightarrow$ no real roots	A1	AO2	
			[7]		
10.	(a)	$(\sqrt{3} - \sqrt{2})^5 = (\sqrt{3})^5 + 5(\sqrt{3})^4(-\sqrt{2})$ $+ 10(\sqrt{3})^3(-\sqrt{2})^2 + 10(\sqrt{3})^2(-\sqrt{2})^3$ $+ 5(\sqrt{3})(-\sqrt{2})^4 + (-\sqrt{2})^5$	B2	AO1 AO1	(five or six terms correct)
		(**If B2 not awarded**, award B1 for three or four correct terms)			
		$(\sqrt{3} - \sqrt{2})^5 = 9\sqrt{3} - 45\sqrt{2} + 60\sqrt{3} - 60\sqrt{2} + 20\sqrt{3} - 4\sqrt{2}$	B2	AO1 AO1	(six terms correct)
		(**If B2 not awarded**, award B1 for three, four or five correct terms)			
		$(\sqrt{3} - \sqrt{2})^5 = 89\sqrt{3} - 109\sqrt{2}$	B1	AO1	(f.t. one error)
	(b)	Since $(\sqrt{3} - \sqrt{2})^5 \approx 0$, we may assume that $89\sqrt{3} \approx 109\sqrt{2}$	M1	AO3	(f.t candidate's answer to part (a) provided one coefficient is negative)
		Either: $89\sqrt{3} \times \sqrt{3} \approx 109\sqrt{2} \times \sqrt{3}$	m1	AO3	(f.t candidate's answer to part (*a*) provided one coefficient is negative)
		$\sqrt{6} \approx \frac{267}{109}$	A1	AO3	(c.a.o.)
		Or $89\sqrt{3} \times \sqrt{2} \approx 109\sqrt{2} \times \sqrt{2}$	(m1)	(AO3)	(f.t candidate's answer to part (*a*) provided one coefficient is negative)
		$\sqrt{6} \approx \frac{218}{89}$	(A1)	(AO3)	(c.a.o.)
			[8]		

Question Number	Solution	Mark	AO	Notes
11.	$a > 0$ $b > a + 2$ $b < 6 + 4a - a^2$	B1 B1 B1 **[3]**	AO1 AO1 AO1	
12.	Let $p = \log_a 19$, $q = \log_7 a$ Then $19 = a^p$, $a = 7^q$	B1	AO2	(the relationship between log and power)
	$19 = a^p = \left(7^q\right)^p = 7^{qp}$	B1	AO2	(the laws of indices)
	$qp = \log_7 19$			(the relationship between log and power)
	$\log_7 a \times \log_a 19 = \log_7 19$	B1 **[3]**	AO2	(convincing)
13. (a)	Choice of variable (x) for $AB \Rightarrow AC = x + 2$	B1	AO3	
	$(x+2)^2 = x^2 + 12^2 - 2 \times x \times 12 \times \frac{2}{3}$	M1	AO3	
	$x^2 + 4x + 4 = x^2 + 144 - 16x$	A1	AO3	
	$20x = 140 \Rightarrow x = 7$ $AB = 7$, $AC = 9$	A1	AO3	(Amend proof for candidates who choose $AC = x$)
(b)	$\sin A\hat{B}C = \frac{\sqrt{5}}{3}$	B1	AO1	
	$\frac{\sin B\hat{A}C}{12} = \frac{\sin A\hat{B}C}{9}$	M1	AO1	f.t. candidate's derived values for AC and $\sin A\hat{B}C$)
	$\sin B\hat{A}C = \frac{4\sqrt{5}}{9}$	A1 **[7]**	AO1	(c.a.o.)
14. (a)	Height of box $= \frac{9000}{2x^2}$	B1	AO3	(o.e.)
	$S = 2 \times (2x \times x + \frac{9000}{2x^2} \times x + \frac{9000}{2x^2} \times 2x$	M1	AO3	(f.t. candidate's derived expression for height of box in terms of x)
	$S = 4x^2 + \frac{27000}{x}$	A1	AO3	(convincing)
(b)	$\frac{dS}{dx} = 8x - \frac{27000}{x^2}$	B1	AO1	
	Putting derived $\frac{dS}{dx} = 0$	M1	AO1	
	$x = 15$	A1	AO1	(f.t. candidate's $\frac{dS}{dx}$)
	Stationary value of S at $x = 15$ is 2700	A1	AO1	(c.a.o)
	A correct method for finding nature of the stationary point yielding a minimum value	B1 **[8]**	AO1	

Question Number		Solution	Mark	AO	Notes
15.	(a)	A represents the initial population of the island.	B1	AO3	
	(b)	$100 = Ae^{2k}$ $160 = Ae^{12k}$	B1	AO1	(both values)
		Dividing to eliminate A	M1	AO1	
		$1{\cdot}6 = e^{10k}$	A1	AO1	
		$k = \frac{1}{10}\ln 1.6 = 0.047$	A1	AO1	(convincing)
	(c)	$A = 91(\cdot 0283)$	B1	AO1	(o.e.)
		When $t = 20$, $N = 91(\cdot 0283) \times e^{0{\cdot}94}$	M1	AO1	(f.t. candidate's derived value for A)
		$N = 233$	A1	AO3	(c.a.o.)
			[8]		
16.		$f'(x) = 3x^2 - 10x - 8$	M1	AO1	(At least one non-zero term correct)
		Critical values $x = -\frac{2}{3}$, $x = 4$	A1	AO1	(c.a.o)
		For an increasing function, $f'(x) > 0$	m1	AO1	
		For an increasing function $x < -\frac{2}{3}$ or $x > 4$	A2	AO2 AO2	(f.t. candidate's derived two critical values for x)
		Deduct 1 mark for each of the following errors the use of non-strict inequalities the use of the word 'and' instead of the word 'or'	**[5]**		

Question Number			Solution	Mark	AO	Notes
17.	(a)		$\frac{\mathrm{d}y}{\mathrm{d}x} = 3 - 2x$	M1	AO1	(At least one non-zero term correct)
			An attempt to find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 2$	m1	AO1	
			At $x = 2$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -1$	A1	AO1	(c.a.o.)
			Equation of tangent at *B* is $y - 2 = -1(x - 2)$	A1	AO1	(f.t. candidate's value for $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 2$)
	(b)		x-coordinate of *A* = 3	B1	AO1	(derived)
			x-coordinate of *C* = 4	B1	AO1	(derived)
			If *D* is the foot of the perpendicular from *B* to the x-axis, area of triangle *BDC* = 2	B1	AO1	(f.t. candidate's derived x-coordinate of *C*)
			Area under curve = $\int_2^3 (3x - x^2)\mathrm{d}x$	M1	AO3	(use of integration) (f.t. candidate's derived x-coordinate of *A*)
			$\frac{3x^2}{2} - \frac{x^3}{3}$	A1	AO3	(correct integration)
			Area under curve = (27/2 – 9) – (6 – 8/3)	m1	AO3	(an attempt to substitute limits, f.t. candidate's derived x-coordinate of *A*)
			Shaded area = Area of triangle *BDC* – Area under curve	m1	AO3	(f.t. candidate's derived x-coordinates of *A* and *C*)
			Shaded area = 5/6	A1	AO3	(c.a.o.)
				[12]		
18.	(a)	(i)	4**u** – 3**v** = 20**i** – 27**j**	B1 B1	AO1 AO1	
		(ii)	A correct method for finding the length of *UV*	M1	AO1	
			Length of *UV* = 10	A1	AO1	
	(b)	(i)	Position vector of $C = \frac{1}{10}\mathbf{a} + \frac{9}{10}\mathbf{b}$ or $C = \frac{9}{10}\mathbf{a} + \frac{1}{10}\mathbf{b}$	M1	AO3	
			Position vector $C = \frac{1}{10}\mathbf{a} + \frac{9}{10}\mathbf{b}$	A1	AO3	
		(ii)	The position vector of any point on the road will be of the form λ**a** + (1 – λ)**b** for some value of λ	B1	AO2	
				[7]		

GCE

MATHEMATICS

UNIT 2: APPLIED MATHEMATICS A

SAMPLE ASSESSMENT MATERIALS

(1 hour 45 minutes)

SECTION A – Statistics

SECTION B – Mechanics

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Take g as 9.8 ms^{-2}.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

SECTION A – Statistics

1. The events A, B are such that $P(A) = 0.2$, $P(B) = 0.3$. Determine the value of $P(A \cup B)$ when

   (a) A, B are mutually exclusive, [2]

   (b) A, B are independent, [3]

   (c) $A \subset B$. [1]

2. Dewi, a candidate in an election, believes that 45% of the electorate intend to vote for him. His agent, however, believes that the support for him is less than this. Given that p denotes the proportion of the electorate intending to vote for Dewi,

   (a) state hypotheses to be used to resolve this difference of opinion. [1]

   They decide to question a random sample of 60 electors. They define the critical region to be $X \leq 20$, where X denotes the number in the sample intending to vote for Dewi.

   (b) (i) Determine the significance level of this critical region.

   (ii) If in fact p is actually 0.35, calculate the probability of a Type II error.

   (iii) Explain in context the meaning of a Type II error.

   (iv) Explain briefly why this test is unsatisfactory. How could it be improved while keeping approximately the same significance level? [8]

3. Cars arrive at random at a toll bridge at a mean rate of 15 per hour.

   (a) Explain briefly why the Poisson distribution could be used to model the number of cars arriving in a particular time interval. [1]

   (b) Phil stands at the bridge for 20 minutes. Determine the probability that he sees exactly 6 cars arrive. [3]

   (c) Using the statistical tables provided, find the time interval (in minutes) for which the probability of more than 10 cars arriving is approximately 0.3. [3]

4. A researcher wishes to investigate the relationship between the amount of carbohydrate and the number of calories in different fruits. He compiles a list of 90 different fruits, e.g. apricots, kiwi fruits, raspberries.

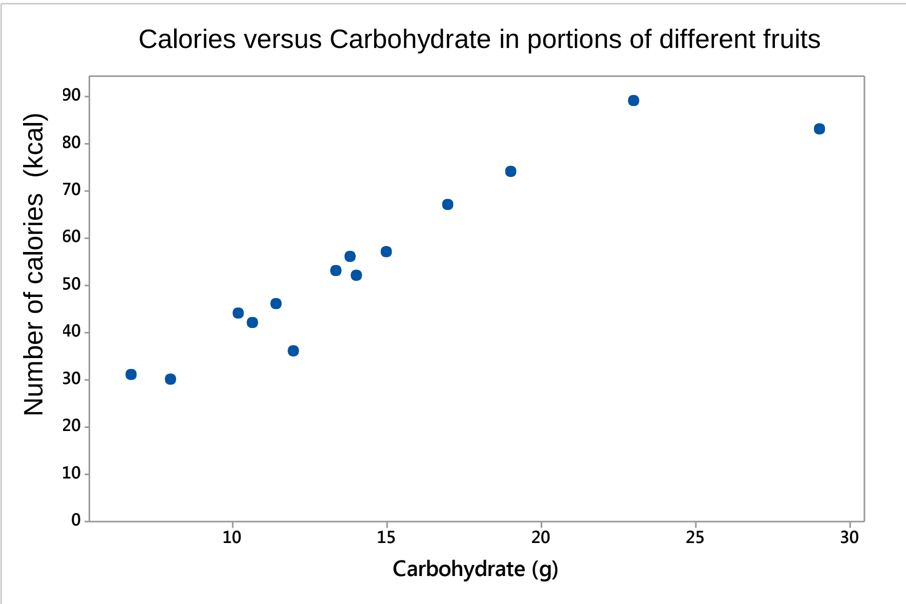
As he does not have enough time to collect data for each of the 90 different fruits, he decides to select a simple random sample of 14 different fruits from the list. For each fruit selected, he then uses a dieting website to find the number of calories (kcal) and the amount of carbohydrate (g) in a typical adult portion (e.g. a whole apple, a bunch of 10 grapes, half a cup of strawberries). He enters these data into a spreadsheet for analysis.

(a) Explain how the random number function on a calculator could be used to select this sample of 14 different fruits. [3]

(b) The scatter graph represents 'Number of calories' against 'Carbohydrate' for the sample of 14 different fruits.

(i) Describe the correlation between 'Number of calories' and 'Carbohydrate'. [1]

(ii) Interpret the correlation between 'Number of calories' and 'Carbohydrate' in this context. [1]



(c) The equation of the regression line for this dataset is:

'Number of calories' = 12.4 + 2.9 × 'Carbohydrate'

(i) Interpret the gradient of the regression line in this context. [1]

(ii) Explain why it is reasonable for the regression line to have a non-zero intercept in this context. [1]

5. Gareth has a keen interest in pop music. He recently read the following claim in a music magazine.

In the pop industry most songs on the radio are not longer than three minutes.

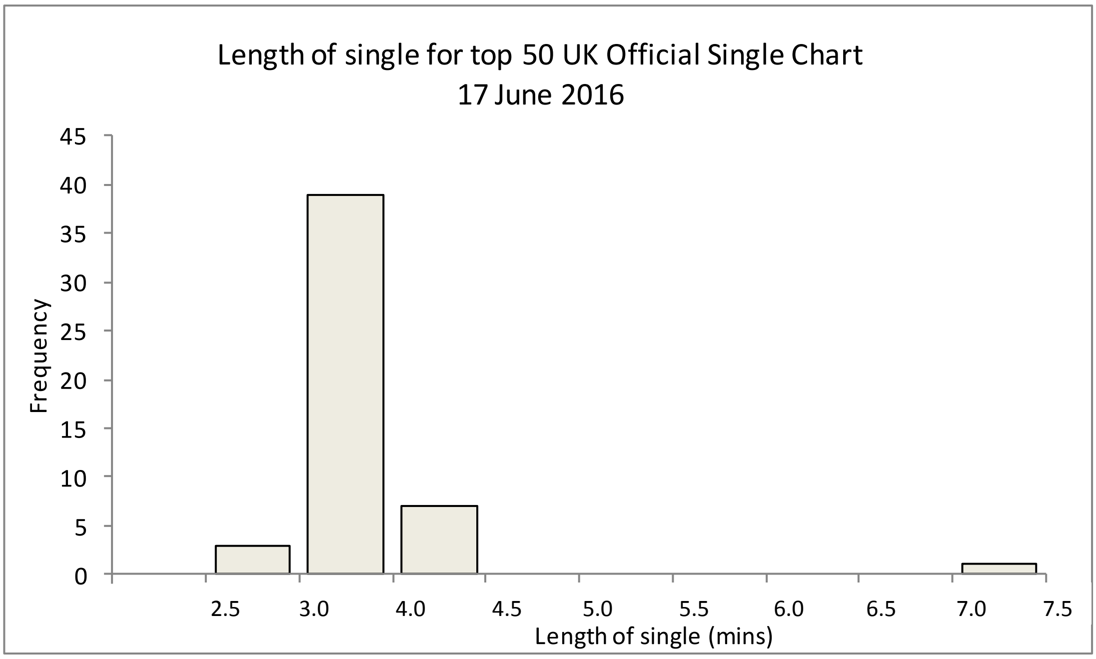
(a) He decided to investigate this claim by recording the lengths of the top 50 singles in the UK Official Singles Chart for the week beginning 17 June 2016. (A 'single' in this context is one digital audio track.)

Comment on the suitability of this sample to investigate the magazine's claim. [1]

(b) Gareth recorded the data in the table below.

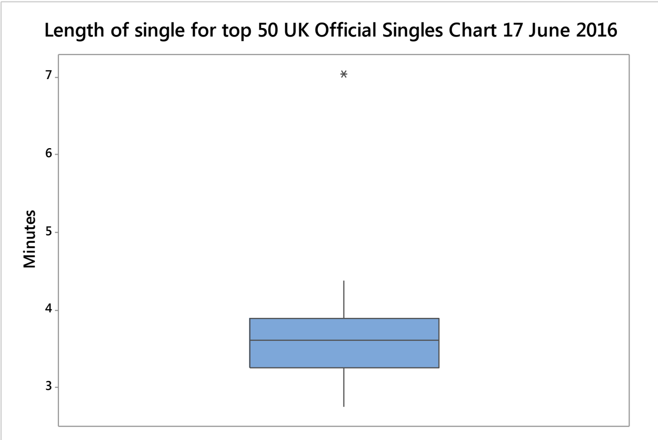
Length of singles for top 50 UK Official Chart singles, 17 June 2016									
2.5–(3.0)	3.0–(3.5)	3.5–(4.0)	4.0–(4.5)	4.5–(5.0)	5.0–(5.5)	5.5–(6.0)	6.0–(6.5)	6.5–(7.0)	7.0–(7.5)
3	17	22	7	0	0	0	0	0	1

He used these data to produce a graph of the distributions of the lengths of singles



State two corrections that Gareth needs to make to the histogram so that it accurately represents the data in the table. [2]

(c) Gareth also produced a box plot of the lengths of singles.



He sees that there is one obvious outlier.

(i) What will happen to the mean if the outlier is removed?

(ii) What will happen to the standard deviation if the outlier is removed? [2]

(d) Gareth decided to remove the outlier. He then produced a table of summary statistics.

(i) Use the appropriate statistics from the table to show, by calculation, that the maximum value for the length of a single is not an outlier.

	Summary statistics Length of single for top 50 UK Official Singles Chart (minutes)							
Length of single	N	Mean	Standard deviation	Minimum	Lower quartile	Median	Upper quartile	Maximum
	49	3.57	0.393	2.77	3.26	3.60	3.89	4.38

(ii) State, with a reason, whether these statistics support the magazine's claim. [4]

(e) Gareth also calculated summary statistics for the lengths of 30 singles selected at random from his personal collection.

	Summary statistics Length of single for Gareth's random sample of 30 singles (minutes)							
Length of single	N	Mean	Standard deviation	Minimum	Lower quartile	Median	Upper quartile	Maximum
	30	3.13	0.364	2.58	2.73	2.92	3.22	3.95

Compare and contrast the distribution of lengths of singles in Gareth's personal collection with the distribution in the top 50 UK Official Singles Chart. [3]

SECTION B – Mechanics

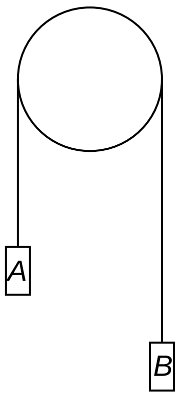
6. A small object, of mass 0.02 kg, is dropped from rest from the top of a building which is160 m high.

   (a) Calculate the speed of the object as it hits the ground. [3]

   (b) Determine the time taken for the object to reach the ground. [3]

   (c) State one assumption you have made in your solution. [1]

7. The diagram below shows two particles *A* and *B*, of mass 2 kg and 5 kg respectively, which are connected by a light inextensible string passing over a fixed smooth pulley. Initially, *B* is held at rest with the string just taut. It is then released.



   (a) Calculate the magnitude of the acceleration of *A* and the tension in the string. [6]

   (b) What assumption does the word 'light' in the description of the string enable you to make in your solution? [1]

8. A particle *P*, of mass 3 kg, moves along the horizontal x-axis under the action of a resultant force F N. Its velocity v ms^{-1} at time t seconds is given by

$$v = 12t - 3t^2.$$

   (a) Given that the particle is at the origin *O* when $t = 1$, find an expression for the displacement of the particle from *O* at time t s. [3]

   (b) Find an expression for the acceleration of the particle at time t s. [2]

9. A truck of mass 180 kg runs on smooth horizontal rails. A light inextensible rope is attached to the front of the truck. The rope runs parallel to the rails until it passes over a light smooth pulley. The rest of the rope hangs down a vertical shaft. When the truck is required to move, a load of M kg is attached to the end of the rope in the shaft and the brakes are then released.

   (a) Find the tension in the rope when the truck and the load move with an acceleration of magnitude 0.8 ms^{-2} and calculate the corresponding value of M. [5]

   (b) In addition to the assumptions given in the question, write down one further assumption that you have made in your solution to this problem and explain how that assumption affects both of your answers. [3]

10. Two forces $\mathbf{F}$ and $\mathbf{G}$ acting on an object are such that

$$\mathbf{F} = \mathbf{i} - 8\mathbf{j},$$
$$\mathbf{G} = 3\mathbf{i} + 11\mathbf{j}.$$

The object has a mass of 3 kg. Calculate the magnitude and direction of the acceleration of the object. [7]

AS Mathematics Unit 2: Applied Mathematics A
General instructions for marking GCE Mathematics

1. The mark scheme should be applied precisely and no departure made from it. Marks should be awarded directly as indicated and no further subdivision made.

2. Marking Abbreviations
The following may be used in marking schemes or in the marking of scripts to indicate reasons for the marks awarded.

cao = correct answer only
MR = misread
PA = premature approximation
bod = benefit of doubt
oe = or equivalent
si = seen or implied
ISW = ignore subsequent working

F.T. = follow through (√ indicates correct working following an error and $\not\surd$ indicates a further error has been made)

Anything given in brackets in the marking scheme is expected but, not required, to gain credit.

3. Premature Approximation
A candidate who approximates prematurely and then proceeds correctly to a final answer loses 1 mark as directed by the Principal Examiner.

4. Misreads
When the data of a question is misread in such a way as not to alter the aim or difficulty of a question, follow through the working and allot marks for the candidates' answers as on the scheme using the new data.
This is only applicable if a wrong value, is used consistently throughout a solution; if the correct value appears anywhere, the solution is not classed as MR (but may, of course, still earn other marks).

5. Marking codes

- ‘M' marks are awarded for any correct method applied to appropriate working, even though a numerical error may be involved. Once earned they cannot be lost.
- ‘m’ marks are dependant method marks. They are only given if the relevant previous ‘M’ mark has been earned.
- ‘A' marks are given for a numerically correct stage, for a correct result or for an answer lying within a specified range. They are only given if the relevant M/m mark has been earned either explicitly or by inference from the correct answer.
- 'B' marks are independent of method and are usually awarded for an accurate result or statement.
- ‘S’ marks are awarded for strategy
- ‘E’ marks are awarded for explanation
- ‘U’ marks are awarded for units
- ‘P’ marks are awarded for plotting points
- ‘C’ marks are awarded for drawing curves

AS Mathematics Unit 2: Applied Mathematics A

Solutions and Mark Scheme

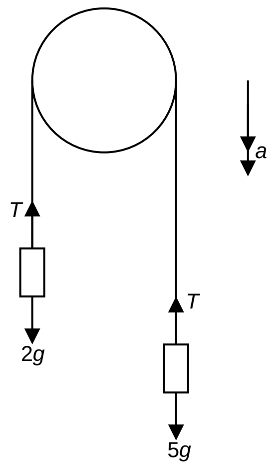
SECTION A – Statistics

Qu. No.	Solution	Mark	AO	Notes
1(a)	$P(A \cup B) = P(A) + P(B)$ $= 0.2 + 0.3 = 0.5$	M1 A1	AO1 AO1	
(b)	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $= P(A) + P(B) - P(A)P(B)$ $= 0.2 + 0.3 - 0.06 = 0.44$	M1 A1 A1	AO1 AO1 AO1	
(c)	$P(A \cup B) = P(B) = 0.3$	B1 **[6]**	AO2	
2(a)	$H_0 : p = 0.45 : H_1 : p < 0.45$	B1	AO3	
(b)(i)	Under H_0, X is B(60,0.45). Sig level = $P(X \le 20)$ $= 0.0446$	B1 M1 A1	AO3 AO2 AO1	
(ii)	Type II error prob = $P(X \ge 21 \mid X$ is B(60,0.35)) $= 0.548$	 M1 A1	 AO2 AO1	
(iii)	A Type II error here is accepting that support for Dewi is 45% when it is actually 35%.	E1	AO3	
(iv)	It is a large value for an error probability. It could be reduced by taking a larger sample.	E1 E1 **[9]**	AO3 AO3	
3(a)	The Poisson distribution can be used when arrivals can be assumed to be independent at a constant mean rate.	E1	AO2	Accept any correct equivalent statement
(b)	The number of arrivals X is Poi(5) $P(X = 6) = \frac{e^{-5} \times 5^6}{6!}$ $= 0.146(22280\ldots)$	B1 M1 A1	AO3 AO1 AO1	Or from the calculator
(c)	Use Poisson tables to find $P(X > 10) = 1 - 0.7060 = 0.2940$ (≈ 0.3) Obtain mean = 9 Therefore time at bridge = 36 minutes	M1 A1 A1 **[7]**	AO3 AO3 AO1	

Qu. No.	Solution	Mark	AO	Notes
4(a)	Allocate each fruit a number 01 to 90 (or 00 to 89)	E1	AO2	
	Generate a random number on a calculator using the random number function.	E1	AO2	
	Match this to the number allocated to the fruits and this is the first member of the sample Repeat this until 14 different fruits are in the sample	E1	AO2	
b)(i)	The correlation is strong and positive.	E1	AO3	
(ii)	More carbohydrates in a fruit suggests more calories.	E1	AO3	
(c)(i)	Each additional gram of carbohydrate corresponds to an increase in the number of calories by 2.9 on average.	E1	AO2	Accept – Each additional gram of carbohydrate corresponds to an increase in the number of calories by 3 on average.
(ii)	If there is no carbohydrate in the fruit there still may be calories present (eg from fat)	E1	AO3	
		[7]		

Qu. No.	Solution	Mark	AO	Notes
5(a)	We cannot be sure that the sample is representative without knowing how the UK Official Singles Chart is constructed.	B1	AO2	Or other valid reason
(b)	Close the gaps between the bars as length of single is a continuous variable	B1	AO3	B0 add gridlines or for any formatting suggestions
	Correct the width of column 3.0–4.0	B1	AO3	
(c)(i)	Mean will decrease	B1	AO2	
(ii)	Standard deviation will decrease	B1	AO2	
(d)(i)	1.5 x (3.89 – 3.26) + 3.89 = 4.84(minutes) Since 4.38(minutes)< 4.84(minutes) not an outlier	M1 A1 B1	AO1 AO1 AO2	
(ii)	Claim is not supported. Median=3.6 > 3 so at least half of singles are longer than 3 mins.	E1	AO3	
(e)	Gareth's singles are shorter than chart singles on average.	E1	AO2	E0 Gareth's singles are shorter
	Gareth's singles are less variable in length than chart singles.	E1	AO2	Or smaller spread
	Chart singles have a roughly symmetrical distribution of lengths, whereas more than half of Gareth's singles are shorter than the mean length.	E1	AO2	Or positively skewed
		[12]		

SECTION B – Mechanics

Question Number	Solution	Mark	AO	Notes
6. (a)	$v^2 = u^2 + 2as$, u=0, a=9.8, s=160 $v^2 = 2 \times 9.8 \times 160$ $v = 56$ (ms^{-1})	M1 A1 A1	AO3 AO1 AO1	
(b)	$s = ut + 0.5at^2$, u=0, a=9.8, s=160 $160 = 0.5 \times 9.8 \times t^2$ $t = \frac{40}{7}$ (s)	M1 A1 A1	AO3 AO1 AO1	
(c)	Object modelled as particle. Air resistance/external forces apart from gravity all ignored.	B1 **[7]**	AO3	
7. (a)				
	Apply N2L to one particle $5g - T = 5a$ Apply N2L to other particle $T - 2g = 2a$ $3g = 7a$ $a = 4.2$ (ms^{-1}) $T = 28$ (N)	M1 A1 A1 m1 A1 A1	AO3 AO2 AO2 AO1 AO1 AO1	
(b)	Light string enables me to assume tension is constant throughout the string.	E1 **[7]**	AO3	

Question Number	Solution	Mark	AO	Notes
8. (a)	$x = \int(12t - 3t^2)\mathrm{d}t$	M1	AO2	
	$x = 6t^2 - t^3 + C$	A1	AO2	correct integration
	When $t = 1, x = 0$			
	$C = -5$			
	$x = 6t^2 - t^3 - 5$	A1	AO2	
(b)	$a = \dfrac{\mathrm{d}v}{\mathrm{d}t}$	M1	AO2	
	$a = 12 - 6t$	A1	AO1	
		[5]		
9. (a)	Apply N2L to truck $T = 180 \times 0.8 = 144$ (N)	B1	AO3	
	Apply N2L to load	M1	AO3	Dimensionally correct eqn T and Mg opposing
	$Mg - T = M \times 0.8$	A1	AO2	
	$M(9.8 - 0.8) = 144$	M1	AO1	substitute value of T
	$M = 16$	A1	AO1	
(b)	No resistance to motion due to external forces, eg air resistance. Truck/load modelled as particle.	B1	AO2	one sensible assumption
	As the truck/load is required to move with acceleration 0.8 ms^{-2}, the value of T would depend on any other external forces. If the resultant external force aids motion, T will be less, but if the external resultant force resists motion, T will be greater.	B1	AO2	any correct statement about T
	The N2L equation will have an extra term opposing motion so M will have to increase.	B1	AO2	any correct statement about M
		[8]		

Question Number	Solution	Mark	AO	Notes
10.	Resultant force vector = $\mathbf{F} + \mathbf{G}$ $= (\mathbf{i} - 8\mathbf{j}) + (3\mathbf{i} + 11\mathbf{j})$ $= 4\mathbf{i} + 3\mathbf{j}$	B1	AO1	
	Magnitude of force = $\sqrt{4^2 + 3^2}$ = 5 (N)	M1 A1	AO1 AO1	
	Use $F = ma$ mag. of acceleration = $\frac{5}{3}$(ms^{-2})	M1 A1	AO3 AO1	
	Let θ be angle direction of motion makes with the vector $\mathbf{i}$. $\tan\theta = \frac{3}{4}$	M1	AO2	
	$\theta = 36.87°$	A1	AO1	
	Alternative solution Resultant force vector = $\mathbf{F} + \mathbf{G}$ $= (\mathbf{i} - 8\mathbf{j}) + (3\mathbf{i} + 11\mathbf{j})$ $= 4\mathbf{i} + 3\mathbf{j}$	(B1)	(AO1)	
	Use $\mathbf{F} = m\mathbf{a}$ $4\mathbf{i} + 3\mathbf{j} = 3\mathbf{a}$	(M1)	(AO3)	
	$\mathbf{a} = \frac{4}{3}\mathbf{i} + \mathbf{j}$	(A1)	(AO1)	
	mag $\mathbf{a} = \sqrt{\left(\frac{4}{3}\right)^2 + 1}$	(M1)	(AO1)	
	mag $\mathbf{a} = \frac{5}{3}$(ms^{-2})	(A1)	(AO1)	
	Direction = $\tan^{-1}\left(\frac{3}{4}\right)$	(M1)	(AO2)	
	$= 36.87°$	(A1)	(AO1)	
		[7]		

GCE

MATHEMATICS

UNIT 3: PURE MATHEMATICS B

SAMPLE ASSESSMENT MATERIALS

(2 hour 30 minutes)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

1. Find a small positive value of x which is an approximate solution of the equation.

$$\cos x - 4\sin x = x^2.$$ [4]

2. Air is pumped into a spherical balloon at the rate of 250 cm^3 per second. When the radius of the balloon is 15 cm, calculate the rate at which the radius is increasing, giving your answer to three decimal places [3]

3. (a) Sketch the graph of $y = x^2 + 6x + 13$, identifying the stationary point. [2]

(b) The function f is defined by $f(x) = x^2 + 6x + 13$ with domain (a, b).

(i) Explain why f^{-1} does not exist when $a = -10$ and $b = 10$. [1]

(ii) Write down a value of a and a value of b for which the inverse of f does exist and derive an expression for $f^{-1}(x)$. [5]

4. (a) Expand $(1-x)^{-\frac{1}{2}}$ in ascending power of x as far as the term in x^2. State the range of x for which the expansion is valid. [2]

(b) By taking $x = \dfrac{1}{10}$, find an approximation for $\sqrt{10}$ in the form $\dfrac{a}{b}$, where a and b are to be determined.

[2]

5. Aled decides to invest £1000 in a savings scheme on the first day of each year. The scheme pays 8% compound interest per annum, and interest is added on the last day of each year. The amount of savings, in pounds, at the end of the third year is given by

$$1000 \times 1\cdot08 + 1000 \times 1\cdot08^2 + 1000 \times 1\cdot08^3$$

Calculate, to the nearest pound, the amount of savings at the end of thirty years. [5]

6. The lengths of the sides of a fifteen-sided plane figure form an arithmetic sequence. The perimeter of the figure is 270 cm and the length of the largest side is eight times that of the smallest side. Find the length of the smallest side. [4]

7. The curve $y = ax^4 + bx^3 + 18x^2$ has a point of inflection at (1, 11).

(a) Show that $2a + b + 6 = 0$. [2]

(b) Find the values of the constants *a* and *b* and show that the curve has another point of inflection at (3, 27). [8]

(c) Sketch the curve, identifying all the stationary points including their nature. [6]

8. (a) Integrate

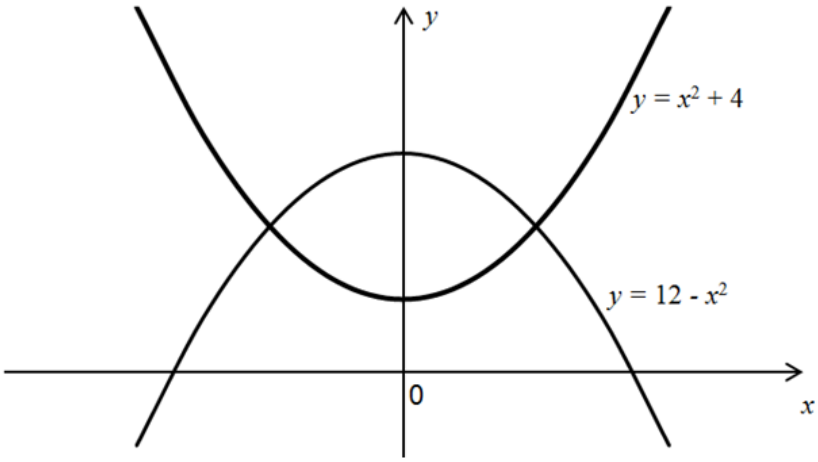
(i) e^{-3x+5} [2]

(ii) $x^2 \ln x$ [4]

(b) Use an appropriate substitution to show that

$$\int_0^{\frac{1}{2}} \frac{x^2}{\sqrt{1-x^2}}\,dx = \frac{\pi}{12} - \frac{\sqrt{3}}{8}.$$ [8]

9.



The diagram above shows a sketch of the curves $y = x^2 + 4$ and $y = 12 - x^2$.

Find the area of the region bounded by the two curves. [6]

10. The equation

$$1+5x-x^4=0$$

has a positive root α.

(a) Show that α lies between 1 and 2. [2]

(b) Use the iterative sequence based on the arrangement

$$x=\sqrt[4]{1+5x}$$

with starting value 1.5 to find α correct to two decimal places. [3]

(c) Use the Newton-Raphson method to find α correct to six decimal places. [6]

11. (a) The curve *C* is given by the equation

$$x^4+x^2y+y^2=13.$$

Find the value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ at the point (-1, 3). [4]

(b) Show that the equation of the normal to the curve $y^2=4x$ at the point $P(p^2,2p)$ is

$$y+px=2p+p^3.$$

Given that $p\neq 0$ and that the normal at *P* cuts the x-axis at $B(b,0)$, show that $b>2$. [7]

12. (a) Differentiate $\cos x$ from first principles. [5]

(b) Differentiate the following with respect to *x*, simplifying your answer as far as possible.

(i) $\dfrac{3x^2}{x^3+1}$ [2]

(ii) $x^3\tan 3x$ [2]

13. (a) Solve the equation

$$\operatorname{cosec}^2 x + \cot^2 x = 5$$

for $0^\circ \le x \le 360^\circ$. [5]

(b) (i) Express $4\sin\theta + 3\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0^\circ \le \alpha \le 90^\circ$. [4]

(ii) Solve the equation

$$4\sin\theta + 3\cos\theta = 2$$

for $0^\circ \le \theta \le 360^\circ$, giving your answer correct to the nearest degree. [3]

14. (a) A cylindrical water tank has base area 4 m^2. The depth of the water at time t seconds is h metres. Water is poured in at the rate 0.004 m^3 per second. Water leaks from a hole in the bottom at a rate of $0.0008h$ m^3 per second. Show that

$$5000\frac{\mathrm{d}h}{\mathrm{d}t} \equiv 5 - h.$$ [2]

[Hint: the volume, V, of the cylindrical water tank is given by $V = 4h$.]

(b) Given that the tank is empty initially, find h in terms of t. [7]

(c) Find the depth of the water in the tank when $t = 3600$ s, giving your answer correct to 2 decimal places. [1]

15. Prove by contradiction the following proposition.

When x is real and positive,

$$4x + \frac{9}{x} \ge 12.$$

The first line of the proof is given below.

Assume that there is a positive and a real value of x such that

$$4x + \frac{9}{x} < 12.$$ [3]

A2 Mathematics Unit 3: Pure Mathematics B
General instructions for marking GCE Mathematics

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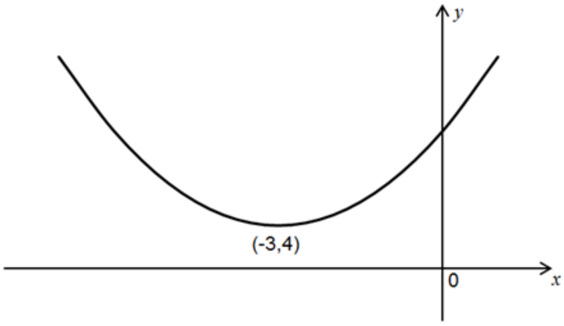
- 'P' marks are awarded for plotting points
- 'C' marks are awarded for drawing curves

A2 Mathematics Unit 3: Pure Mathematics B

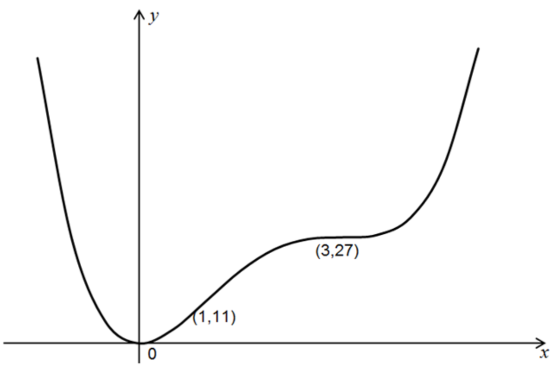
Solutions and Mark Scheme

Question Number	Solution	Mark	AO	Notes
1. (a)	$1-\frac{x^2}{2}-4x=x^2$	M1	AO1	(Attempt to substitute for $\cos x, \sin x$)
	$\frac{3x^2}{2}+4x-1=0$	A1	AO1	(Correct)
	$3x^2+8x-2=0$	B1	AO1	
	$x=\frac{-8\pm\sqrt{64+24}}{6}=\frac{-8\pm\sqrt{88}}{6}$			
	$x=0.230(1385...)\,,\,(-2.896805...)$	B1	AO1	
		[4]		
2.	$V=\frac{4}{3}\pi r^3$			
	$\frac{\mathrm{d}V}{\mathrm{d}t}=3\times\frac{4}{3}\pi r^2\frac{\mathrm{d}r}{\mathrm{d}t}$	B1	AO3	
	$4\pi\times15^2\frac{\mathrm{d}r}{\mathrm{d}t}=250$	M1	AO3	(Substitution of data)
	$\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{250}{900\pi}\approx0.088\,(\text{cm/second})$	A1	AO3	
		[3]		

Question Number	Solution	Mark	AO	Notes
3. (a)		G1 G1	AO1 AO1	(Shape) (Stationary point)
(b) (i)	A correct statement, eg. f^{-1} doesn't exist because f is not a one-one function	E1	AO2	
(ii)	Any appropriate domain eg. There are many possible appropriate domains. It is essential that any domain must be contained in one branch of the curve shown.	B1	AO2	
	Here we consider $(-3, \infty)$. Let $y = x^2 + 6x + 13$ $= (x+3)^2 + 4$ $x + 3 = \pm\sqrt{y-4}$	M1	AO1	(Attempt to find x in terms of y)
	So that $x = -3 \pm \sqrt{y-4}$	A1	AO1	
	Since $x > -3$, the positive sign is appropriate $\therefore x = -3 + \sqrt{y-4}$	A1	AO2	
	And $f^{-1}(x) = -3 + \sqrt{x-4}$	A1	AO2	
		[8]		

Question Number	Solution	Mark	AO	Notes
4. (a)	$(1-x)^{-\frac{1}{2}} = 1 + \frac{x}{2} + \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\frac{x^2}{2} + ..$			
	$= 1 + \frac{x}{2} + \frac{3x^2}{8} + ...$	B1	AO1	
	Valid for $\lvert x \rvert < 1$	B1	AO1	
	When $x = \frac{1}{10}$, $\left(\frac{9}{10}\right)^{-\frac{1}{2}} \approx 1 + \frac{1}{20} + \frac{3}{800} = \frac{843}{800}$	B1	AO2	
	So that $(10)^{\frac{1}{2}} = 3\text{x}\frac{843}{800} = \frac{2529}{800}$	B1	AO1	
		[4]		
5.	After 30 years, saving is			
	$(1.08)1000 + (1.08)^2 1000 + ... + (1.08)^{30} 1000$	B1	AO3	
	This is G.P with $a = (1\cdot08)1000$			
	$r = 1\cdot08$			
	and $n = 30$	B2	AO3,AO3	(B2 for 3 correct, B1 for 2 correct)
	Then			
	$S_{30} = (1000)(1.08)\left(\frac{(1.08)^{30} - 1}{0.08}\right)$	M1	AO3	(correct formula)
	$\approx £122,346$	A1	AO3	
		[5]		

Question Number	Solution	Mark	AO	Notes
6.	If smallest side is a, largest side $= 8a$			
	$8a = a + 14d$ $a = 2d$	M1 A1	AO3 AO3	(Attempt to relate the two sides)
	Perimeter $= \frac{15}{2}\left[2a + 14d\right] = \frac{15}{2}.18d = 135d$	M1	AO3	
	$\therefore 135d = 270$ $d = 2$ Length of smallest side $= a = 2d = 4$ cm	B1	AO3	
	Alternative mark scheme: smallest side = a, largest side $= 8a$			
	Perimeter $= \frac{15}{2}\left[a + 8a\right] = \frac{15}{2}.9a = \frac{135}{2}a$	(M1) (A1)	(AO3) (AO3)	
	$\therefore \frac{135}{2}a = 270$	(M1)	(AO3)	
	$a = 4$ Length of smallest side $= a = 4$ cm	(A1) **[4]**	(AO3)	

Question Number		Solution	Mark	AO	Notes
7.	(a)	$\frac{d^2y}{dx^2} = 12ax^2 + 6bx + 36$	M1	AO2	(attempt to find $\frac{d^2y}{dx^2}$, 2 correct terms)
		For point of inflection at $(1,11)$ $12a + 6b + 36 = 0$ So that $2a + b + 6 = 0$ (1)	A1	AO2	
	(b)	Also $a + b + 18 = 11$ (2)	B1 M1	AO1 AO1	(Attempt to solve for a, b)
		From (1), (2), $a = 1$, $b = -8$	A1	AO1	
		$\therefore \frac{d^2y}{dx^2} = 12x^2 - 48x + 36$ $= 12(x^2 - 4x + 3) = 12(x-1)(x-3) = 0$	M1	AO2	
		$\therefore \frac{d^2y}{dx^2} = 0$ when $x = 3$	A1	AO2	
		and $\frac{d^2y}{dx^2}$ changes sign as x passes through 3	m1	AO2	
		$\therefore$ There is a point of inflection at $x = 3$, $y = 3^4 - 8.3^3 + 18.3^2 = 27$, i.e at $(3,27)$	A1 A1	AO2 AO2	(Only if m1 is awarded)
	(c)	$\frac{dy}{dx} = 4x^3 - 24x^2 + 36x = 0$	M2	AO1,AO1	(M1 for correct differentiation but not equal to 0)
		$\therefore 4x(x^2 - 6x + 9) = 0$ giving $x = 0, x = 3$	A1	AO1	(point of Inflection) (Two Values)
		Then at $x = 0$, $y = 0$ and $\frac{d^2y}{dx^2} = 36$ There is a minimum at $x = 0, y = 0$	A1	AO1	
			G1 G1	AO1 AO1	general shape min two points of inflection
			[16]		

Question Number	Solution	Mark	AO	Notes
8 (a) (i)	$-\dfrac{e^{-3x+5}}{3}+C$	M1 A1	AO1 AO1	($ke^{-3x+5)}$) ($k=-\dfrac{1}{3}$)
(ii)	$\int x^2 \ln x \, dx$ $u=\ln x, \dfrac{dv}{dx}=x^2$ $\dfrac{du}{dx}=\dfrac{1}{x}, \ v=\dfrac{x^3}{3}$ $\int x^2 \ln x \, dx = \dfrac{x^3}{3}\ln x - \int \dfrac{x^3}{3}.\dfrac{1}{x} dx$ $=\dfrac{x^3}{3}\ln x - \dfrac{x^3}{9}+C$ (Penalise omission of C once only)	M1 A1,A1 A1	AO1 AO1, AO1 AO1	(Correct u and $\dfrac{dv}{dx}$)
(b)	$\int_0^{\frac{1}{2}} \dfrac{x^2}{\sqrt{1-x^2}} dx$ $x=\sin\theta \quad dx=\cos\theta \, d\theta$ $x=0, \ \theta=0 \quad x=\dfrac{1}{2}, \ \theta=\dfrac{\pi}{6}$ $=\int_0^{\frac{\pi}{6}} \dfrac{\sin^2\theta\cos\theta}{\sqrt{1-\sin^2\theta}} d\theta$ $=\int_0^{\frac{\pi}{6}} \dfrac{\sin^2\theta}{\cos\theta}\cos\theta \, d\theta$ $=\int_0^{\frac{\pi}{6}} \sin^2\theta \, d\theta$ $=\int_0^{\frac{\pi}{6}} \dfrac{1-\cos 2\theta}{2} d\theta$ $=\left[\dfrac{\theta}{2}-\dfrac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{6}}$ $\dfrac{\pi}{12}-\dfrac{\sin\frac{\pi}{3}}{4}-0+0=\dfrac{\pi}{12}-\dfrac{\sqrt{3}}{8}$	B1 B1 M1 A1 A1 m1 A1 A1 **[14]**	AO3 AO3 AO3 AO3 AO3 AO3 AO3 AO3	 (attempt to substitute) (Correct) (both correct)

Question Number	Solution	Mark	AO	Notes
9.	$x^2 + 4 = 12 - x^2$ $2x^2 = 8$ $x = \pm 2$	M1 A1	AO3 AO3	(Equating y's)
	Area $= \int_{-2}^{2} \{12 - x^2 - (x^2 + 4)\}\ \mathrm{d}x$	M1	AO3	(expressing area)
	$= \int_{-2}^{2} (8 - 2x^2)\mathrm{d}x$			
	$= \left[8x - \dfrac{2x^3}{3} \right]_{-2}^{2}$	A2	AO3 AO3	(F.T arithmetic error)
	$= \dfrac{64}{3}$	A1	AO3	(c.a.o)
	Alternative mark scheme for the Area:			
	Area $= \int_{-2}^{2} (12 - x^2)\mathrm{d}x - \int_{-2}^{2} (x^2 + 4)\,\mathrm{d}x$	(M1)	(AO3)	
	$= \left[12x - \dfrac{x^3}{3} - \dfrac{x^3}{3} - 4x \right]_{-2}^{2}$	(A2)	(AO3) (AO3)	(A2 for 4 terms correct, A1 for 2 terms correct)
	$= \dfrac{64}{3}$	(A1) **[6]**	(AO3)	(c.a.o)

Question Number	Solution	Mark	AO	Notes
10. (a)	$f(x) = 1 + 5x - x^4$ $f(1) = 5,\ f(2) = -5$	M1	AO2	(Use of Intermediate Value Theorem.)
	There is a change of sign indicating there is a root between 1 and 2.	A1	AO2	(correct values and conclusions)
(b)	$x_{n+1} = \sqrt[4]{1+5x_n},\quad x_0 = 1\cdot 5,\quad x_1 = 1.707476485$	B1	AO1	
	$x_2 = 1.75734609$	B1	AO1	
	$x_3 = 1.7687213,\quad x_4 = 1.7712854$			
	$x_5 = 1.771861948,\ \alpha \approx 1.77$	B1	AO1	
(c)	$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} = x_n - \dfrac{1+5x_n - x_n^4}{5-4x_n^3}$	M1	AO1	Attempt to use Newton-Raphson
		A1	AO1	All terms correct
	$x_0 = 1.5$			
	$x_1 = 1.904411765$	M1 A1	AO1 AO1	
	$x_2 = 1.788115338$			
	$x_3 = 1.772305156$			
	$x_4 = 1.772029085$			
	$x_5 = 1.772028972$	A1	AO1	
	Root $\alpha \approx 1.772029$	A1	AO1	Correct to 6 decimal places
		[11]		

Question Number	Solution	Mark	AO	Notes
11. (a)	$4x^3 + 2xy + x^2 \frac{dy}{dx} + 2y\frac{dy}{dx} = 0$	B2	AO1,AO1	(B2, 4 correct terms) (B1, 3 correct terms)
	Now, $x = -1, y = 3$ so that $-4 - 6 + \frac{dy}{dx} + 6\frac{dy}{dx} = 0$	B1	AO1	
	$\frac{dy}{dx} = \frac{10}{7}$	B1	AO1	
(b)	$\frac{dy}{dx} = \frac{dy}{dp} / \frac{dx}{dp} = \frac{2}{2p} = \frac{1}{p}$	M1 A1	AO1 AO1	
	Gradient of normal is $-p$	B1	AO1	
	Equation of normal is $(y - 2p) = -p(x - p^2)$	m1	AO1	
	$y - 2p = -px + p^3$			
	so that $y + px = 2p + p^3$	A1	AO1	convincing
	When $y = 0, \quad x = b$ $b = 2 + p^2$	B1	AO2	
	Since $p^2 > 0,\ b > 2$	E1	AO2	
		[11]		

Question Number	Solution	Mark	AO	Notes
12. (a)	Let $y = \cos x$			
	$\frac{dy}{dx} = \lim_{h \to 0}\left[\frac{\cos(x+h) - \cos x}{h}\right]$	M1	AO2	
	$= \lim_{h \to 0}\left[\frac{\cos x \cos h - \sin x \sin h - \cos x}{h}\right]$	A1	AO2	
	As h approaches 0 $\cos h \approx 1 - \frac{h^2}{2}$ and $\sin h \approx h$			
	So $\frac{dy}{dx} = \lim_{h \to 0}\left[\frac{\cos x\left(1 - \frac{h^2}{2}\right) - \sin x \times h - \cos x}{h}\right]$	M1	AO2	
	$= \lim_{h \to 0}\left[\frac{-\frac{h^2}{2}\cos x - h\sin x}{h}\right]$	A1	AO2	
	$= -\sin x$	A1	AO2	
(b) (i)	$\frac{(x^3+1)6x - 3x^2(3x^2)}{(x^3+1)^2}$	M1	AO1	(Correct formula)
	$= \frac{3x(2-x^3)}{(x^3+1)^2}$	A1	AO1	
(ii)	$3x^2 \tan 3x + 3x^3 \sec^2 3x$	M1	AO1	(Correct formula)
	$= 3x^2(\tan 3x + x\sec^2 3x)$	A1	AO1	(All Correct)
		[9]		

Question Number	Solution	Mark	AO	Notes
13. (a)	$\operatorname{cosec}^2 x + \cot^2 x = 5$			
	$1 + 2\cot^2 x = 5$	M1	AO1	(Attempt to write in terms of one function)
	$\cot^2 x = 2$	A1	AO1	
	$\tan x = \pm\dfrac{1}{\sqrt{2}}$	A1	AO1	
	$x = 35.3,\ 215.3^\circ, 144.7^\circ,\ 324.7^\circ$	B1,B1	AO1 AO1	(each pair)
(b) (i)	$4\sin\theta + 3\cos\theta \equiv R(\sin\theta\cos\alpha + \cos\theta\sin\alpha)$			
	$R\cos\alpha = 4$ $R\sin\alpha = 3$	B1 B1	AO1 AO1	
	$R = \sqrt{3^2 + 4^2} = 5$	B1	AO1	
	$\tan\alpha = \dfrac{3}{4},\quad \alpha = 36.87^\circ$	B1	AO1	
	$4\sin\theta + 3\cos\theta \equiv 5\sin(\theta + 36.87^\circ)$			
(ii)	$5\sin(\theta + 36.87^\circ) = 2$			
	$\sin(\theta + 36.87^\circ) = 0.4$	B1	AO1	
	$\theta + 36.87^\circ = 23.58^\circ, 156.42^\circ, 383.58^\circ$			
	$\theta = 119.5(5)^\circ,\ 346.7(1)^\circ$ $= 120^\circ, 347^\circ$ to the nearest degree	B1 B1	AO1 AO1	
		[12]		

Question Number	Solution	Mark	AO	Notes
14. (a)	$\dfrac{\mathrm{d}V}{\mathrm{d}t} = 4\dfrac{\mathrm{d}h}{\mathrm{d}t}$			
	$4\dfrac{\mathrm{d}h}{\mathrm{d}t} = 0.004 - 0.0008h$	M1	AO3	(3 terms, at least 2 correct)
	$\dfrac{\mathrm{d}h}{\mathrm{d}t} = 0.001 - 0.0002h$			
	$5000\dfrac{\mathrm{d}h}{\mathrm{d}t} = 5 - h$	A1	AO3	(Correct)
(b)	$5000\int \dfrac{\mathrm{d}h}{5-h} = \int \mathrm{d}t$	M1	AO1	(Separation of variables)
	$-5000\ \ln(5-h) = t + C$ (1)	A1,A1	AO1 AO1	(-1 if C omitted)
	$h = 0$ at $t = 0$			
	$\therefore -5000\ln(5) = C$	m1	AO1	
	Substitute in (1)			
	$-5000\ \ln(5-h) = t - 5000\ln(5)$			
	$t = 5000\ln\left(\dfrac{5}{5-h}\right)$	A1	AO1	
	$\therefore \left(\dfrac{5}{5-h}\right) = \mathrm{e}^{\frac{t}{5000}}$	M1	AO1	(Attempt to invert)
	$5 - h = 5\mathrm{e}^{\frac{-t}{5000}}$			
	$h = 5 - 5\mathrm{e}^{\frac{-t}{5000}}$	A1	AO1	
(c)	$h = 5 - 5\mathrm{e}^{\frac{-3600}{5000}}$			
	$= 2.57\,\mathrm{m}$	B1	AO1	
		[10]		

Question Number	Solution	Mark	AO	Notes
15.	$4x^2 + 9 < 12x$ $4x^2 - 12x + 9 < 0$	M1	AO2	(Clear fractions)
	$(2x-3)^2 < 0$ Impossible when x is real. Contradiction so that assumption is false.	A1	AO2	
	$\therefore 4x + \dfrac{9}{x} \geq 12$	A1	AO2	
		[3]		

GCE

MATHEMATICS

UNIT 4: APPLIED MATHEMATICS B

SAMPLE ASSESSMENT MATERIALS

(1 hour 45 minutes)

SECTION A – Statistics

SECTION B – Differential Equations and Mechanics

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Take g as 9.8 ms^{-2}.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

SECTION A – Statistics

1. It is known that 4% of a population suffer from a certain disease. When a diagnostic test is applied to a person with the disease, it gives a positive response with probability 0.98. When the test is applied to a person who does not have the disease, it gives a positive response with probability 0.01.

   (a) Using a tree diagram, or otherwise, show that the probability of a person who does not have the disease giving a negative response is 0.9504. [2]

   The test is applied to a randomly selected member of the population.

   (b) Find the probability that a positive response is obtained. [2]

   (c) Given that a positive response is obtained, find the probability that the person has the disease. [2]

2. Mary and Jeff are archers and one morning they play the following game. They shoot an arrow at a target alternately, starting with Mary. The winner is the first to hit the target. You may assume that, with each shot, Mary has a probability 0.25 of hitting the target and Jeff has a probability p of hitting the target. Successive shots are independent.

   (a) Determine the probability that Jeff wins the game

   i) with his first shot,

   ii) with his second shot. [4]

   (b) Show that the probability that Jeff wins the game is [3]

$$\frac{3p}{1+3p}$$

   (c) Find the range of values of p for which Mary is more likely to win the game than Jeff. [2]

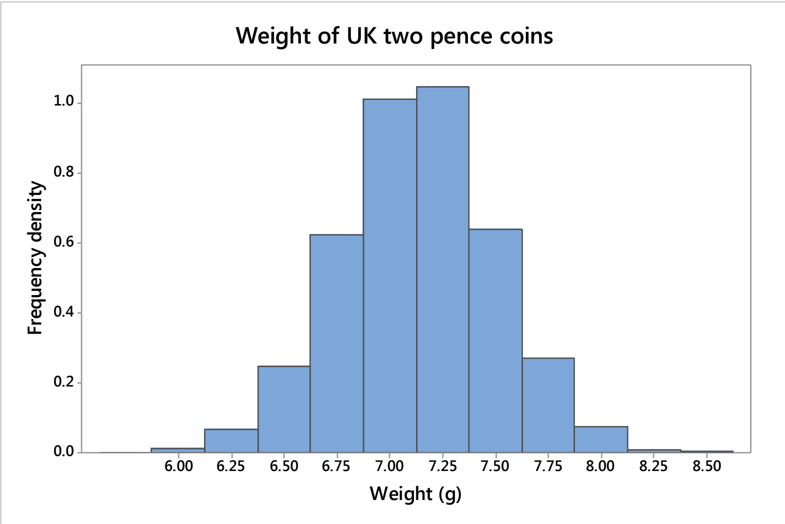
3. A string of length 60 cm is cut a random point.

   (a) Name a distribution, including parameters, that can be used to model the length of the longer piece of string and find its mean and variance. [3]

   (b) The longer string is shaped to form the perimeter of a circle. Find the probability that the area of the circle is greater than 100 cm^2. [4]

4. Automatic coin counting machines sort, count and batch coins. A particular brand of these machines rejects 2p coins that are less than 6.12 grams or greater than 8.12 grams.

(a) The histogram represents the distribution of the weight of UK 2p coins supplied by the Royal Mint. This distribution has mean 7.12 grams and standard deviation 0.357 grams.



Explain why the weight of 2p coins can be modelled using a normal distribution. [1]

(b) Assume the distribution of the weight of 2p coins is normally distributed. Calculate the proportion of 2p coins that are rejected by this brand of coin counting machine. [2]

(c) A manager suspects that a large batch of 2p coins is counterfeit. A random sample of 30 of the suspect coins is selected. Each of the coins in the sample is weighed. The results are shown in the summary statistics table.

Summary statistics Weights (in grams) for a random sample of 30 UK 2p coins						
Mean	Standard deviation	Minimum	Lower quartile	Median	Upper quartile	Maximum
6.89	0.296	6.45	6.63	6.88	7.08	7.48

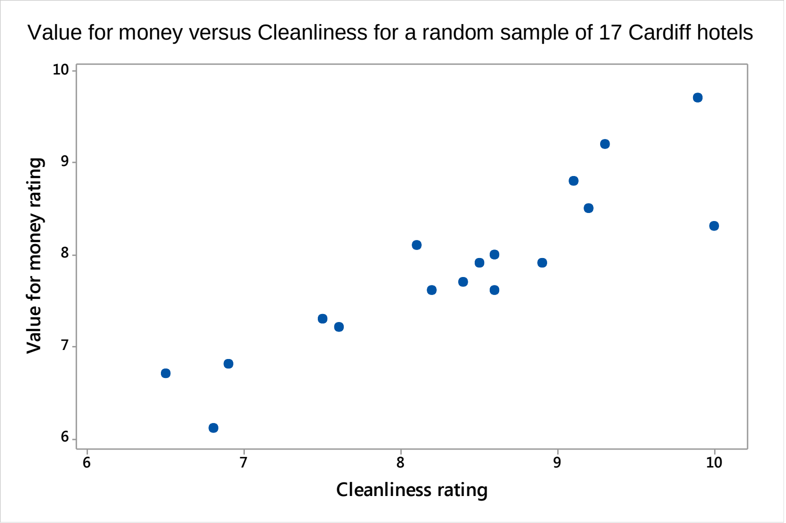
i) What assumption must be made about the weights of coins in this batch in order to conduct a test of significance on the sample mean? State, with a reason, whether you think this assumption is reasonable. [2]

ii) Assuming the population standard deviation is 0.357 grams, test at the 1% significance level whether the mean weight of the 2p coins in this batch is less than 7.12 grams. [6]

5. A hotel owner in Cardiff is interested in what factors hotel guests think are important when staying at a hotel. From a hotel booking website he collects the ratings for 'Cleanliness', 'Location', 'Comfort' and 'Value for money' for a random sample of 17 Cardiff hotels.

(Each rating is the average of all scores awarded by guests who have contributed reviews using a scale from 1 to 10, where 10 is 'Excellent'.)

The scatter graph shows the relationship between 'Value for money' and 'Cleanliness' for the sample of Cardiff hotels.



(a) The product moment correlation coefficient for 'Value for money' and 'Cleanliness' for the sample of 17 Cardiff hotels is 0.895.
Stating your hypotheses clearly, test, at the 5% level of significance, whether this correlation is significant. State your conclusion in context. [5]

(b) The hotel owner also wishes to investigate whether 'Value for money' has a significant correlation with 'Cost per night'. He used a statistical analysis package which provided the following output which includes the Pearson correlation coefficient of interest and the corresponding *p*-value.

	Value for money	Cost per night
Value for money	1	
Cost per night	0.047 (0.859)	1

Comment on the correlation between 'Value for money' and 'Cost per night'. [2]

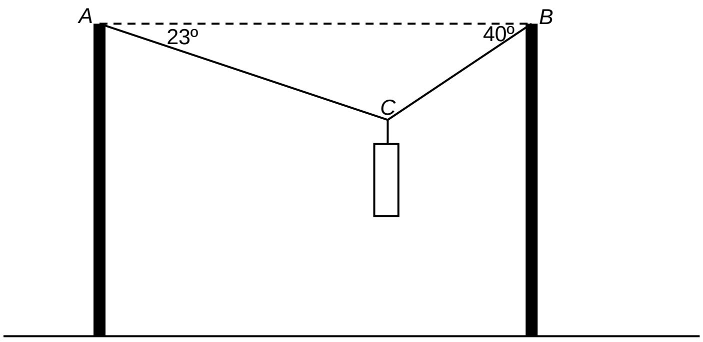
SECTION B – Differential Equations and Mechanics

6. An object of mass 4 kg is moving on a horizontal plane under the action of a constant force $4\mathbf{i} - 12\mathbf{j}$ N. At time $t = 0$ s, its position vector is $7\mathbf{i} - 26\mathbf{j}$ with respect to the origin *O* and its velocity vector is $-\mathbf{i} + 4\mathbf{j}$.

   (a) Determine the velocity vector of the object at time $t = 5$ s. [3]

   (b) Calculate the distance of the object from the origin when $t = 2$ s. [5]

7. The diagram below shows an object of weight 160 N at a point *C*, supported by two cables *AC* and *BC* inclined at angles of 23º and 40º to the horizontal respectively.



   (a) Find the tension in *AC* and the tension in *BC*. [6]

   (b) State two modelling assumptions you have made in your solution. [2]

8. The rate of change of a population of a colony of bacteria is proportional to the size of the population P, with constant of proportionality k. At time $t = 0$ (hours), the size of the population is 10.

   (a) Find an expression, in terms of k, for P at time t. [6]

   (b) Given that the population doubles after 1 hour, find the time required for the population to reach 1 million. [3]

9. A particle of mass 12 kg lies on a rough horizontal surface. The coefficient of friction between the particle and the surface is 0.8. The particle is at rest. It is then subjected to a horizontal tractive force of magnitude 75 N. Determine the magnitude of the frictional force acting on the particle, giving a reason for your answer. [5]

10. A body is projected at time $t = 0$ s from a point O with speed V ms^{-1} in a direction inclined at an angle of θ to the horizontal.

(a) Write down expressions for the horizontal and vertical components x m and y m of its displacement from O at time t s. [2]

(b) Show that the range R m on a horizontal plane through the point of projection is given by

$$R = \frac{V^2}{g}\sin 2\theta$$

[3]

(c) Given that the maximum range is 392 m, find, correct to one decimal place,

i) the speed of projection,

ii) the time of flight,

iii) the maximum height attained. [5]

A2 Mathematics Unit 4: Applied Mathematics B
General instructions for marking GCE Mathematics

1. The mark scheme should be applied precisely and no departure made from it. Marks should be awarded directly as indicated and no further subdivision made.

2. Marking Abbreviations
The following may be used in marking schemes or in the marking of scripts to indicate reasons for the marks awarded.

cao = correct answer only
MR = misread
PA = premature approximation
bod = benefit of doubt
oe = or equivalent
si = seen or implied
ISW = ignore subsequent working

F.T. = follow through (✓ indicates correct working following an error and ✗ indicates a further error has been made)

Anything given in brackets in the marking scheme is expected but, not required, to gain credit.

3. Premature Approximation
A candidate who approximates prematurely and then proceeds correctly to a final answer loses 1 mark as directed by the Principal Examiner.

4. Misreads
When the data of a question is misread in such a way as not to alter the aim or difficulty of a question, follow through the working and allot marks for the candidates' answers as on the scheme using the new data.
This is only applicable if a wrong value, is used consistently throughout a solution; if the correct value appears anywhere, the solution is not classed as MR (but may, of course, still earn other marks).

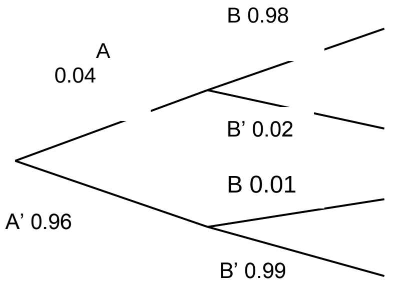
5. Marking codes

- 'M' marks are awarded for any correct method applied to appropriate working, even though a numerical error may be involved. Once earned they cannot be lost.
- 'm' marks are dependant method marks. They are only given if the relevant previous 'M' mark has been earned.
- 'A' marks are given for a numerically correct stage, for a correct result or for an answer lying within a specified range. They are only given if the relevant M/m mark has been earned either explicitly or by inference from the correct answer.
- 'B' marks are independent of method and are usually awarded for an accurate result or statement.
- 'S' marks are awarded for strategy
- 'E' marks are awarded for explanation
- 'U' marks are awarded for units
- 'P' marks are awarded for plotting points
- 'C' marks are awarded for drawing curves

A2 Mathematics Unit 4: Applied Mathematics B

Solutions and Mark Scheme

<u>SECTION A – Statistics</u>

Qu. No.	Solution	Mark	AO	Notes
1(a)		M1	AO1	diagram
	A = the event that a person has the disease. B = the event that a positive response is obtained			
	Prob = $0.96 \times 0.99 = 0.9504$	A1	AO2	
	Alternative mark scheme for (a):			
	Prob = 0.96×0.99 $= 0.9504$	(M1) (A1)	(AO1) (AO2)	
(b)	$P(B) = 0.04 \times 0.98 + 0.96 \times 0.01$ $= 0.0488$	M1 A1	AO3 AO1	
(c)	$P(A\|B) = \dfrac{P(A \cap B)}{P(B)}$			
	$= \dfrac{0.04 \times 0.98}{0.0488}$	M1	AO3	
	$= 0.803(278688\ldots)$	A1	AO1	
		[6]		

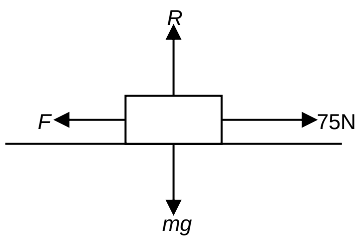
Qu. No.	Solution	Mark	AO	Notes
2(a)(i)	P(J wins with 1st shot) =P(M misses) $\times$ P(J hits) $= 0.75p$	M1 A1	AO1 AO1	
(ii)	J wins with his second shot if the first three shots miss and then J hits the target with his second shot. P(J wins with 2nd shot) = $0.75 \times (1 - p) \times 0.75 \times p$	M1 A1	AO3 AO2	
(b)	P(J wins game) = $0.75p + 0.75^2(1-p)p + 0.75^3(1-p)^2p + ...$ Attempting to sum an infinite geometric series $= \dfrac{0.75p}{1-0.75(1-p)}$ $= \dfrac{3p}{1+3p}$	M1 M1 A1	AO3 AO3 AO2	
(c)	Mary is more likely to win if $\dfrac{3p}{1+3p} < 0.5$ leading to $p < \dfrac{1}{3}$	M1 A1 **[9]**	AO3 AO1	
3(a)	Continuous uniform distribution on [30,60] Mean = 45 Variance = 75	B1 B1 B1	AO3 AO1 AO1	
(b)	$P(\pi R^2 > 100) = P\left(R > \sqrt{\dfrac{100}{\pi}}\right)$ $= P\left(L > 2\pi\sqrt{\dfrac{100}{\pi}}\right)$ $= P(L > 35.45)$ $= \dfrac{60-35.45}{30} = 0.818(\dot{3})$ or $\dfrac{491}{600}$	M1 A1 A1 A1 **[7]**	AO3 AO2 AO1 AO1	

Qu. No.	Solution	Mark	AO	Notes
4(a)	Bell shaped	B1	AO2	Or Most values cluster in the middle of the range and the rest taper off symmetrically toward either extreme B0 for symmetrical only
(b)	1- P(6.12 < X < 8.12) = 1- 0.9949(0744) = 0.0051 (or 0.51%)	M1 A1	AO3 AO1	Or P(X < 6.12) + P(X > 8.12) M1A0 For 0.9949(0744)
(c)(i)	The population of weights of 2p coins is normally distributed. Mean and median in the sample are very similar, suggesting a symmetric distribution.	B1 B1	AO2 AO2	B1B0 The weights of 2p coins are normally distributed. Population must be stated or implied.
(ii)	H_o: The mean weight of all 2p coins in this batch = 7.12g H_1: The mean weight of all 2p coins in this batch < 7.12g (one-sided)	B1	AO3	Or H_o: μ= 7.12g B0 for H_o: Mean = 7.12g Population must be stated or implied, ie. the batch of 2p coins
	p-value = P($\bar{x} < 6.89 \mid \mathrm{H_0}$) $= \mathrm{P}\left(z < \frac{6.89-7.12}{\frac{0.357}{\sqrt{30}}}\right)$	M1	AO1	FT two-sided test
	$= \mathrm{P}(z < -3.52(874))$ = 0.00021 (allow 0.00022) Since *p*-value<0.01, Reject H_o	A1 A1 A1	AO1 AO1 AO2	*p*-value = 2 × 0.00021 = 0.00042
	Very strong evidence to suggest the mean weight of the batch of 2p coins is less than 7.12(g)	E1	AO3	
	Alternative Solution: TS = $\frac{6.89-7.12}{\frac{0.357}{\sqrt{30}}}$ = -3.52(874) CV = -2.32(63) Since TS< CV Reject H_o	 (M1) (A1) (A1) (A1)	 (AO1) (AO1) (AO1) (AO2)	 FT Two-sided test CVs = ± 2.576 Since TS< - 2.576
	Very strong evidence to suggest the mean weight of the batch of 2p coins is less than 7.12(g)	(E1) **[11]**	(AO3)	

Qu. No.	Solution	Mark	AO	Notes
5(a)	H_o: $\rho = 0$ H_1: $\rho \neq 0$ two-sided	B1	AO3	H_o: $\rho = 0$ H_1: $\rho > 0$ one-sided Population stated or implied
	TS = 0.895	B1	AO1	TS = 0.895
	CV = ±0.4821	B1	AO1	CV = ±0.412
	Since TS>0.4821, Reject H_o	B1	AO2	Since TS>0.412, Reject H_o
	Strong evidence to suggest the correlation coefficient is greater than zero	E1	AO3	Strong evidence to suggest the correlation coefficient is greater than zero
(b)	P-value for correlation between Value for money and Cost per night is > 0.05	E1	AO2	
	Cost per night does not seem to be correlated to Value for money.	E1	AO2	
		[7]		

SECTION B – Differential Equations and Mechanics

Question Number	Solution	Mark	AO	Notes
6. (a)	$\mathbf{a} = \mathbf{F}/m = \frac{1}{4}(4\mathbf{i} - 12\mathbf{j})$	M1	AO3	
	$\mathbf{a} = \mathbf{i} - 3\mathbf{j}$			
	Use $\mathbf{v} = \mathbf{u} + \mathbf{a}t$, $\mathbf{u} = -\mathbf{i} + 4\mathbf{j}$, $\mathbf{a} = \mathbf{i} - 3\mathbf{j}$	M1	AO2	
	$\mathbf{v} = (-\mathbf{i} + 4\mathbf{j}) + 5(\mathbf{i} - 3\mathbf{j})$			
	$\mathbf{v} = 4\mathbf{i} - 11\mathbf{j}$	A1	AO1	
(b)	$s = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2 + 7\mathbf{i} - 26\mathbf{j}$	M1	AO2	position vector relative to initial position vector.
		m1	AO2	adding initial positionvector.
	$\mathbf{s} = 2(-\mathbf{i} + 4\mathbf{j}) + \frac{1}{2} \times 4 \times (\mathbf{i} - 3\mathbf{j}) + (7\mathbf{i} - 26\mathbf{j})$			
	$\mathbf{s} = 7\mathbf{i} - 24\mathbf{j}$	A1	AO1	
	$\lvert\mathbf{s}\rvert = \sqrt{7^2 + 24^2}$	m1	AO1	
	$\lvert\mathbf{s}\rvert = 25$	A1	AO1	
		[8]		
7. (a)	Attempt to resolve in 2 directions	M1	AO3	dimensionally correct equation, no omitted or extra forces
	$T_1 \cos 23^\circ = T_2 \cos 40^\circ$	A1	AO2	correct equation
	$T_1 \sin 23^\circ + T_2 \sin 40^\circ = 160$	A1	AO2	correct equation
	Attempt to solve simultaneously	m1	AO1	any valid method
	$T_1 = 137.56(028\ldots)$ (N)	A1	AO1	
	$T_2 = 165.29(707\ldots)$ (N)	A1	AO1	
(b)	Object modelled as particle	B1	AO3	
	Cable modelled as light strings	B1	AO3	
		[8]		

Question Number	Solution	Mark	AO	Notes
8. (a)	$\frac{\mathrm{d}P}{\mathrm{d}t} = kP$	M1	AO3	
	$\int \frac{\mathrm{d}P}{P} = \int k\, dt$	m1	AO2	separation of variables
	$\ln P = kt + C$	A1	AO1	correct integration
	when $t = 0$, $P = 10$			
	$C = \ln 10$	m1	AO2	
	$\ln \frac{P}{10} = kt$			
	$e^{kt} = \frac{P}{10}$	m1	AO2	
	$P = 10\, e^{kt}$	A1	AO1	
(b)	When $t = 1$, $P = 20$			
	$k = \ln 2$	M1	AO2	
	$t = \frac{\ln 0 \cdot 1P}{\ln 2}$			
	When $P = 1000000$			
	$t = \frac{\ln 100000}{\ln 2}$	m1	AO1	
	$t = 16.61$ hours	A1	AO1	
		[9]		
9.				
	$R = mg = 12 \times 9.8$ (= 117.6 N)	B1	AO1	
	Maximum friction = μR	M1	AO3	used
	Maximum friction = $0.8 \times 12 \times 9.8$ (= 94.08N)	A1	AO1	
	Therefore frictional force = 75 (N)	B1	AO3	
	because Max friction > tractive force	E1	AO3	
		[5]		

Question Number	Solution	Mark	AO	Notes
10. (a)	$x = (V\cos\theta)t$	B1	AO1	
	$y = (V\sin\theta)t - \frac{1}{2}gt^2$	B1	AO1	
(b)	$y = 0$ for time of flight $t = \dfrac{2V\sin\theta}{g}$	M1	AO2	
	Range $R = V\cos\theta . \dfrac{2V\sin\theta}{g}$	m1	AO2	
	$R = \dfrac{V^2\sin 2\theta}{g}$	A1	AO2	
(c) (i)	At maximum range, $\sin 2\theta = 1$ $\theta = 45°$ $\dfrac{V^2}{g} = 392$	M1	AO3	oe
	$V = 62.0$ (ms^{-1})	A1	AO1	cao
(ii)	$t = \dfrac{2\times 62\cdot 0\times\sin 45}{g}$ $t = 8.95$ (s)	A1	AO1	cao
(iii)	Max height when t =4.47 s, $y_{max} = 62.5\times\sin 45°\times 4.47 - \frac{1}{2}\times 9.8\times 4.47^2$	m1	AO2	
	$y_{max} = 98.1$ (m)	A1	AO1	cao
		[10]		

APPENDIX

ASSESSMENT OBJECTIVE WEIGHTINGS

GCE MATHEMATICS

Level	AO1	AO2	AO3	TOTAL
AS	93	51	51	195
	48%	26%	26%	
Total mark for assessment objectives must be in the range	88 - 107	39 - 58	39 - 58	
	(45% - 55%)	(20% - 30%)	(20% - 30%)	

Level	AO1	AO2	AO3	TOTAL
A2	102	48	50	200
	51%	24%	25%	
Total mark for assessment objectives must be in the range	90 - 110	40 - 60	40 - 60	
	(45% - 55%)	(20% - 30%)	(20% - 30%)	

Level	AO1	AO2	AO3	TOTAL
A LEVEL	195	99	101	395
	49%	25%	26%	
Total mark for assessment objectives must be in the range	178 - 217	79 - 118	79 - 118	
	(45% - 55%)	(20% - 30%)	(20% - 30%)	

AS Mathematics Unit 1: Pure Mathematics A (120 marks)

Question Number	AO1	AO2	AO3	TOTAL
1	7	0	0	**7**
2	6	0	0	**6**
3	0	6	0	**6**
4	0	0	5	**5**
5	10	2	0	**12**
6	0	5	0	**5**
7	3	2	0	**5**
8	0	1	5	**6**
9	4	3	0	**7**
10	5	0	3	**8**
11	3	0	0	**3**
12	0	3	0	**3**
13	3	0	4	**7**
14	5	0	3	**8**
15	6	0	2	**8**
16	3	2	0	**5**
17	7	0	5	**12**
18	4	1	2	**7**
TOTAL	**66**	**25**	**29**	**120**
Total mark for assessment objectives must be in the range	62 - 73	21 - 32	21 - 32	

AS Mathematics Unit 2: Applied Mathematics A (75 marks)

Question Number	AO1	AO2	AO3	TOTAL
1	5	1	0	**6**
2	2	2	5	**9**
3	3	1	3	**7**
4	0	4	3	**7**
5	2	7	3	**12**
6	4	0	3	**7**
7	3	2	2	**7**
8	1	4	0	**5**
9	2	4	2	**8**
10	5	1	1	**7**
				0
				0
				0
				0
TOTAL	**27**	**26**	**22**	**75**
Total mark for assessment objectives must be in the range	26 - 34	19 - 26	19 - 26	

A2 Mathematics Unit 3: Pure Mathematics B (120 marks)

Question Number	AO1	AO2	AO3	TOTAL
1	4	0	0	**4**
2	0	0	3	**3**
3	4	4	0	**8**
4	3	1	0	**4**
5	0	0	5	**5**
6	0	0	4	**4**
7	9	7	0	**16**
8	6	0	8	**14**
9	0	0	6	**6**
10	9	2	0	**11**
11	9	2	0	**11**
12	4	5	0	**9**
13	12	0	0	**12**
14	8	0	2	**10**
15	0	3	0	**3**
				0
				0
				0
				0
TOTAL	**68**	**24**	**28**	**120**
Total mark for assessment objectives must be in the range	63 - 74	20 - 31	20 - 31	

A2 Mathematics Unit 4: Applied Mathematics B (80 marks)

Question Number	AO1	AO2	AO3	TOTAL
1	3	1	2	**6**
2	3	2	4	**9**
3	4	1	2	**7**
4	4	4	3	**11**
5	2	3	2	**7**
6	4	3	1	**8**
7	3	2	3	**8**
8	4	4	1	**9**
9	2	0	3	**5**
10	5	4	1	**10**
				0
				0
				0
				0
TOTAL	**34**	**24**	**22**	**80**
Total mark for assessment objectives must be in the range	28 - 36	20 - 28	20 - 28	