



GCE EXAMINERS' REPORTS

**GCE
MATHEMATICS
AS/Advanced**

SUMMER 2023

Introduction

Our Principal examiners' reports offer valuable feedback on the recent assessment series. They are written by our Principal Examiners and Principal Moderators after the completion of marking and moderation, and detail how candidates have performed.

This report offers an overall summary of candidates' performance, including the assessment objectives/skills/topics/themes being tested, and highlights the characteristics of successful performance and where performance could be improved. It goes on to look in detail at each question/section of each unit, pinpointing aspects that proved challenging to some candidates and suggesting some reasons as to why that might be.ⁱ

The information found in this report can provide invaluable insight for practitioners to support their teaching and learning activity. We would also encourage practitioners to share this document – in its entirety or in part – with their learners to help with exam preparation, to understand how to avoid pitfalls and to add to their revision toolbox.

Further support

Document	Description	Link
Professional Learning / CPD	WJEC offers an extensive annual programme of online and face-to-face Professional Learning events. Access interactive feedback, review example candidate responses, gain practical ideas for the classroom and put questions to our dedicated team by registering for one of our events here.	https://www.wjec.co.uk/home/professional-learning/
Past papers	Access the bank of past papers for this qualification, including the most recent assessments. Please note that we do not make past papers available on the public website until 6 months after the examination.	www.wjecservices.co.uk or on the WJEC subject page
Grade boundary information	<p>Grade boundaries are the minimum number of marks needed to achieve each grade.</p> <p>For unitised specifications grade boundaries are expressed on a Uniform Mark Scale (UMS). UMS grade boundaries remain the same every year as the range of UMS mark percentages allocated to a particular grade does not change. UMS grade boundaries are published at overall subject and unit level.</p> <p>For linear specifications, a single grade is awarded for the overall subject, rather than for each unit that contributes towards the overall grade. Grade boundaries are published on results day.</p>	For unitised specifications click here: Results, Grade Boundaries and PRS (wjec.co.uk)

Exam Results Analysis	WJEC provides information to examination centres via the WJEC secure website. This is restricted to centre staff only. Access is granted to centre staff by the Examinations Officer at the centre.	www.wjecservices.co.uk
Classroom Resources	Access our extensive range of FREE classroom resources, including blended learning materials, exam walk-throughs and knowledge organisers to support teaching and learning.	https://resources.wjec.co.uk/
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Subject Officer's Executive Summary

The Summer 2023 GCE Mathematics AS and A2 papers were generally accessible to all candidates, with an appropriate balance of differentiating questions to challenge the most able candidates.

Overall, candidates performed better in the Summer 2023 examinations, relative to the Summer 2022 series. However, poor algebraic skills, coupled with careless arithmetic errors, often cost candidates valuable marks.

Other areas for improvement are as follows:

Areas for improvement	Classroom resources	Brief description of resource
Sketching graphs	Geogebra	Free software package – for teaching and learning mathematics (and science).
Conditional probability	Knowledge Organisers	Worked examples of information represented in a variety of forms.
Differential equations	Knowledge Organisers	Notes on rates of change, proportionality, and construction of differential equations.

MATHEMATICS
General Certificate of Education
Summer 2023
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PURE MATHEMATICS A – AS UNIT 1

Overview of the Unit

This paper seemed to have been well received by candidates. Excellent solutions were seen to all 14 questions. Even for question 7, which proved difficult for a number of candidates, many excellent full-mark solutions were seen.

The general opinion among examiners was that this paper was relatively straightforward, with very few marks that were only obtainable by the most able candidates. This was supported by the better performance generally by candidates, compared to the summer 2022 paper.

All topics seemed to have been reasonably well understood by many candidates. Generally speaking, candidates are weak at sketching graphs, even very standard ones. The use of graphical calculators does not seem to be widespread. In addition, many candidates' performance could well be improved if they took the time to draw reasonably accurate diagrams. A relatively accurate diagram would enhance their understanding of the requirements of the question.

Comments on individual questions/sections

- Q.1 This question was well done by the majority of candidates. Disappointingly, in part (a), far too many candidates failed to bracket the term $(-3x)^2$ and obtained the incorrect third term. Some candidates lost the negative sign all together.
- Q.3 This question was well done until part (d). Often the absence of a reasonably accurate diagram meant that candidates identified the wrong diameter for the circle. Spending a little time to draw an accurate diagram is probably time well spent.
- Q.7 For part (a) of this question, there were two methods that could be used. Both methods involved an initial step of equating the two given equations. Candidates could then either find the discriminant, or equate the gradient function to m , the gradient of the given line. Although the second method is far easier than the first method, the most popular method was the discriminant approach. It was unusual to see a completely correct solution. Many candidates were unable to answer (i), but they were able to use the equation given in (i) to write a completely correct answer to (ii); or they were able to answer (i) well, but they failed to answer (ii). In many solutions, work appropriate to (i) was seen in (ii).

Inexplicably, solutions for part (b) were disappointing. Many candidates did not attempt part (b), or, if they did, they were unable to determine the correct regions.

Q.9 Part (a) was a question on differentiation from first principles. As this was a proof question, it was marked very strictly. Most candidates did not gain the final two quality marks, as often, there were some errors in their presentation, e.g. incorrect notation.

In (b)(ii), candidates needed to solve the inequality $6x^{\frac{1}{2}} - 3x^{-\frac{3}{2}} > 0$. The algebraic manipulation required with the fractional indices proved too difficult for most candidates and there were very few correct solutions seen.

Q.10 In part (c), the usual errors were seen when applying the laws of logarithms. In particular, when candidates attempted to combine the second and third terms, they failed to spot the negative sign in front of the second term and used the addition law rather than the subtraction law. Many candidates who got all the laws correct were unable to deal with $\log_3 81$, not realising that this could be evaluated using a calculator. Also, many candidates failed to solve the resulting simple log equation $\log_7 x^2 = 4$, which was disappointing.

Q.11 In part (a), some very strange and incorrect graphs were seen. Candidates often lost the curve in the second quadrant. Otherwise, this was a generally well-done question.

In part (b), most candidates correctly translated their graph to the right. Some forgot to indicate how many units to the right and candidates also frequently lost the B1 mark for stating the point where the curve crossed the y -axis.

In part (c), some candidates' sketches were not sufficiently accurate and, as a result, they missed one of the points of intersection in the first quadrant.

Q.12 It was very disappointing that candidates were unable to recall the definition of a unit vector, particularly since this also featured in the summer 2022 Unit 1 paper. However, this did not prevent candidates from accessing subsequent parts of the question, and it was pleasing to see that part (c) was generally well done.

Q.13 This question was generally well done. Although candidates were able to apply the correct method in answering the question, they often lost marks due to arithmetic errors.

Q.14 Very few candidates were able to successfully answer part (a), as many candidates did not seem to understand what was required.

Part (b) was generally well done by candidates who realised that $A = 5$. A few were able to derive that from the information given in the question. Many candidates lost the final mark for failing to subtract 4 hours from their value of t , obtained for a concentration of 0.6. However, many candidates who did not realise that A was the initial concentration were not able to make progress on this question.

Q.2, Q.4, Q.5, Q.6, Q.8 – There were no areas to highlight on these questions.

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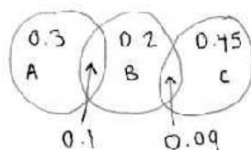
APPLIED MATHEMATICS A – AS UNIT 2 SECTION A

Overview of the Unit

With examinations resuming a certain normality, the residual impact of the loss of learning through covid is still being felt, with some general principles of mathematics not as internalised by candidates as expected at this level. Candidates seemed more prepared for some questions, than for others, leading to a wide range of accessibility to individual questions. Compared to previous series, some questions were answered well. Candidates seemed to have most difficulty with recognising the discrete uniform distribution, and working with a discrete probability distribution. Candidates seemed to have most success with the binomial and Poisson distributions and interpreting data.

Comments on individual questions/sections

- Q.1 A gentle start to the paper, where the vast majority of candidates were able to answer parts (a) and (b) successfully. In part (c), a common error was to not recognise how the three circles overlapped. Candidates who had drawn three overlapping circles were required to label the relevant sections that had a probability of 0. Candidates had most difficulty labelling the areas $P(A \cap B')$ and $P(C \cap B')$, which they confused with $P(A)$ and $P(C)$ respectively, leading to diagrams like the one shown. A box was required for completeness.



Candidates are encouraged to check that the probabilities in their Venn diagrams sum to 1. A variety of methods were used in part (d), with, of course, the simple addition of the probabilities on the Venn diagram being the most straightforward. The stronger candidates recognised this, whereas the weaker candidates used the addition formula. The weakest candidates were not able to successfully answer this part. The addition formula was helpful for those who were not able to get a correct Venn diagram. This was the most successfully answered question in Section A.

- Q.2 Candidates, for the most part, seemed to only have knowledge of two distributions: the binomial distribution and the Poisson distribution. Candidates should also be familiar with the discrete uniform distribution. This was the most poorly answered question on the paper.

- Q.3 This was the least attempted question on the paper. Despite being a staple of the legacy specification, discrete probability distributions have not yet been assessed in this manner in the reformed specification and this was evident from candidates' performance on this question. Going forward, candidates should, hopefully, be better equipped in answering questions of this type in future series.
- Q.4 This question was answered reasonably well. Candidates are encouraged to write the distribution that they are using and any helpful equations like $P(X \geq 10) = 1 - P(X \leq 9)$. Not writing these led to all sorts of errors. Candidates are also encouraged to understand that calculators will give $P(X \leq x)$. Common errors included finding $P(X = 10)$, $1 - (X \leq 10)$ and using $Po(4)$.

In part (b), candidates who were able to recognise the binomial distribution did well. Unfortunately, many candidates were not able to recognise the binomial distribution and continued using a Poisson distribution.

- Q.5 Candidates are getting better at answering hypothesis testing questions. There were far fewer instances where candidates simply stated ' $40 \div 156 = 25.6\%$ which is less than 34% so reject H_0 '. This is encouraging. This question was, on the whole, answered relatively successfully, despite some candidates still calculating $P(X = 40)$ in part (b)(ii); this scored no further marks. Candidates still struggle to explain concepts like a p -value, or a test statistic, in context.
- Q.6 Despite the relative accessibility of part (a), a disproportionately high number of candidates were unable to reject 752 and 195. Part (b) allowed candidates to think for themselves and, encouragingly, many were able to give sensible reasons. Part (c) was also answered well, although a common error was to make statements based on numbers of people, rather than percentages. It was disappointing to see that many candidates were unable to perform a simple calculation to find the standard deviation. In part (d)(ii), candidates are encouraged to remember to refer to 'on average' or 'in general' when making comparisons. Comparisons in questions like this should be stated more contextually and should state more than simply, 'the mean is higher for children who speak Welsh'.

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APPLIED MATHEMATICS A – AS UNIT 2 SECTION B

Overview of the Unit

The paper allowed candidates of all abilities to display their knowledge and demonstrate their mathematical skills. The attempt rates were consistently high, supporting the fact that there was sufficient time to answer Section B of the paper. Furthermore, there were no stand-out facility factors for Section B, suggesting that all questions were generally accessible to most candidates. There were very few marks that were only obtainable for the most able candidates. As usual, many exemplar solutions were seen for all of the questions in Section B.

Comments on individual questions/sections

- Q.7 This question on ‘lifts’ was well done by the vast majority of candidates. It was encouraging to see that most candidates considered a suitable sketch, with forces and directions clearly labelled. Very few candidates opted to work in terms of g , but this was not necessary. Poor algebraic manipulation caused some problems in part (b), and some did not correctly identify the normal reaction when considering the forces on the person in isolation. Instead, the normal reaction was incorrectly taken as the tension in part (a).
- Q.8 This question was reasonably well understood by many candidates. Some concluded with $v = -16$ and so lost the final mark, which required candidates to state the speed. A misconception in this question was to find out when the speed of the particle was zero.
- Q.9 This question performed very well overall, with a variety of successful methods being seen. The most common errors were due to algebraic slips using the constant k , with the method applied being correct. The most successful candidates considered a simple sketch to help determine the angle in part (b). Disappointingly, many candidates did not opt to use the ‘more reliable’ 17, that was provided in the question, in their trigonometric ratio.
- Q.10 Very few candidates managed to secure the one mark for part (a). Unfortunately, many incorrectly stated that the tension in the string connecting the objects was zero. Finding the acceleration in part (b)(i) was done very well by the vast majority of candidates, as expected. Attempts for part (b)(ii) were also very good, with most candidates using their acceleration from part (i). The most frequent error was to assume that the particles travel a distance of 1.8 m in order to be at the same height.

Q.11 Parts (a) and (b) were generally answered well, demonstrating that most candidates are comfortable with the properties of velocity-time graphs. Attempts in part (c) were less successful since this related to the 'negative portion' the graph. Part (d) was designed to test understanding by challenging candidates to think about velocity and displacement simultaneously. Many excellent scripts were seen in which this part was, unfortunately, answered incorrectly. Similarly, some weaker candidates secured the mark, suggesting that some candidates may have simply guessed the correct region.

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PURE MATHEMATICS B – A2 UNIT 3

Overview of the Unit

Candidates performed reasonably well on this paper. Excellent full-mark solutions were seen to all 14 questions. The first 3 questions were very well done generally and provided a good start to the paper. Not surprisingly, the additional formulae given in 2023, together with those given in the Formula Booklet, resulted in the majority of candidates achieving full marks on question 1. Question 9(a) proved difficult; candidates were mostly let down by poor algebra skills.

The general opinion among examiners was that this was a fair paper with a good proportion of accessible marks, as well as questions designed to differentiate, for the most able candidates. This was supported by the overall better performance of candidates, compared to the summer 2022 paper.

All topics seemed to be reasonably well understood by many candidates. Generally speaking, as in Unit 1, candidates did not perform well at sketching graphs, with the graphs of $y = |2x + 3|$ and $y = |5x - 3|$ often being incorrect. In addition, some candidates' presentation of solutions was so poorly organised that it often became difficult to be certain whether candidates had used correct valid methods to answer some questions.

Comments on individual questions/sections

Q.4 In part (a), some candidates worked out $f(-1)$ and $f(1)$, noted that both were positive, but only a few candidates checked a value between -1 and 1 . It should, however, have been apparent that $f(0) = -6$. Other candidates simply solved the cubic equation on their calculators and found the 2 roots that lie in the interval $-1 < x < 1$.

In part (c), the marks were awarded for knowing that the formula fails at the second iteration and stating why this was the case. If candidates had just carried out the iterations, the answer would have become obvious. Unfortunately, many candidates simply said that, because there was a square root in the formula, it would fail as you cannot take the square root of a negative number, i.e. $x_0 = -3$. This was not awarded any marks.

Q.5 This question required candidates to translate the information given in the question into mathematics. Questions of this type, i.e. set in context, are usually poorly answered and this was no exception. Many candidates did not arrive at the correct geometric series, but the confusion with arithmetic progressions was not often seen, as was the case in 2022. The most common errors were using 80 or 112 as the first term, rather than the correct value of 32.

Q.7 This question involved two very simple modulus functions and many candidates managed to produce a reasonable sketch, though a mark was often lost as the second point of intersection (0, 3) was not correctly shown. Disappointingly, when given the point of intersection (2, 7), candidates were not able to reflect this information in their sketch. This consequently affected candidates' ability to correctly answer part (c), as the required triangle was not correctly reflected in their sketch, and so they were not able to find its area. Many candidates assumed incorrectly that the triangle was right-angled. Some candidates used a less efficient method by finding all three sides of the triangle, then finding an angle using the cosine rule, and finally using $\text{Area} = \frac{1}{2} ab \sin C$. Others integrated to find the area of the trapezium and the triangles, which was also a less efficient method.

Q.9 In part (a), candidates lost marks due to incorrect algebraic manipulation, as they tried to arrive at the expression given in the question. The presentation of candidates' solutions was sometimes very poorly organised, as candidates would simply provide a list of expressions, without any indication with regards to what the expressions represented.

Part (b) was generally well done and the expected error with candidates using 5 or 7 ordinates was seen. Many candidates did not arrive at the correct final answer even though all the terms were worked out correctly, which was a pity. Part (c) was well done, and some nice illustrative diagrams were seen.

Q.12 Candidates made the usual errors when separating variables and not remembering to include the constant of integration.

Q.13 Unfortunately, many candidates went into auto pilot and found $\frac{dx}{dp}$, $\frac{dy}{dp}$, then $\frac{dy}{dx}$, which was irrelevant and earned no marks.

Q.14 Part (b) involved integration by substitution. As the substitution, which was given in the question, was relatively straightforward, part (b) was reasonably well answered, though most candidates lost the final mark for omitting the modulus signs, or the constant of integration, or for not rewriting the expression in u in terms of x .

Q.1, Q.2, Q.3, Q.6, Q.8, Q.10, Q.11 – There were no issues to highlight on these questions.

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APPLIED MATHEMATICS B – A2 UNIT 4 SECTION A

Overview of the Unit

This paper certainly had challenging elements. Candidates did not perform as well as expected on this unit. Candidates found question 2, in particular, rather challenging. Despite this, it was evident that some candidates were struggling with the mathematical knowledge and understanding required at this level. Although understanding of probability was particularly poor, some candidates performed well on questions assessing hypothesis tests.

Comments on individual questions/sections

- Q.1 Part (a) proved to be a gentle start to the paper with many candidates performing well. All but the weakest candidates were able to pick up at least one mark in part (a). In part (b), candidates seemed perplexed by the fact that one region in the Venn diagram was within another. Most candidates had a typical two-circle, overlapping Venn diagram and often had a non-zero value for the section 'less than 4 but not less than 8'. A box was required for completeness. Most candidates were able to pick up at least one mark in part (b) too.
- Q.2 This question certainly proved challenging for many candidates. It was the most poorly answered question on the paper, by a considerable margin. Candidates who were able to draw a tree diagram were at an advantage over those who did not. Despite the question proving challenging for candidates, many were able to pick up some marks. Fully correct solutions were not particularly common. Some candidates were able to show that $p = 0.09$, but not from the requisite equation to find d . Others were able to form a correct equation in p and d , and were only able to calculate $d = 0.74$ by using the information given in the question. Forming the initial equation in p and d proved difficult for many candidates. Failing to deal with $(0.96 - p) \times 0.02$ was the most common error. When showing $p = 0.09$, many candidates were able to substitute the correct value for the denominator and the probability $\frac{87}{610}$ given in the question into the conditional probability equation; however, once again, candidates were unable to deal with the term $(0.96 - p) \times 0.02$. Those who had attempted to form an equation earlier in the question were credited for substituting 0.09 into the equation, even if the equation was incorrect.
- Q.3 This question proved challenging for candidates because the distribution did not start at 0. In part (a), many candidates arrived at an answer of $22.\dot{1}$ from $\frac{19}{d-1} = 0.9$, or $21.\dot{1}$ from $\frac{19}{d} = 0.9$. Fortunately, there were follow-through marks available in part (b), although it was a requirement that $d > 19$. In part (b), since the question asked for the standard deviation, candidates who gave their final answer as the variance did not earn the relevant mark.

- Q.4 Part (b) was answered considerably more successfully than part (a). Many candidates seemed unsure on how to approach part (a). Those that were able to answer the question did well. Some common errors included using 0.8416 instead of -0.8416 and setting the standardised expression to 0.11 or 0.2. The majority of candidates attempted part (b) and a variety of methods were used.
- Q.5 This question was generally well answered. Some common errors included stating incorrect hypotheses, not labelling the test statistic and an incorrect final conclusion for the hypotheses stated. There were some very insightful answers from candidates in part (b), including 'medical treatment for the disease by 2020 has improved'.

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APPLIED MATHEMATICS B – A2 UNIT 4 SECTION B

Overview of the Unit

The paper allowed candidates of all abilities to display their knowledge and demonstrate their mathematical skills. The attempt rates were consistently high, supporting the fact that there was sufficient time to answer Section B of the paper. All questions were generally accessible to most candidates, with the exception of question 10, which had a disappointing facility factor of 28.5. There were very few marks that were only obtainable for the most able candidates. Section B in the Summer 2022 paper appears to have been more accessible than the Summer 2022 paper and, as usual, many exemplar solutions were seen for all of the questions.

Comments on individual questions/sections

Q.6 There is no area to highlight.

Q.7 A variety of different responses were seen for this ‘unfamiliar’ question on moments. Unfortunately, many candidates did not heed the instruction in the question and consequently assumed that the rod was uniform. As expected, this approach did not earn full credit. Remarkably, a significant number of candidates still opted for two applications of moments, which caused problems in identifying the unknown distance. Invariably, this approach was much less successful than resolving forces vertically, coupled with a single application of moments.

A significant number of candidates were less successful in securing the final mark in part (b), as many incorrectly concluded that the centre of mass lying at the midpoint of the rod implied that the rod was uniform.

Q.8 This was the most successful question on the entire paper, and it was reassuring to see that candidates were not overly troubled by the context of this question. Sadly, only a handful of candidates recognised that the minimum speed was $23 \cos 18^\circ > 21$, since there was no horizontal acceleration in the underlying model. Instead, in the vast majority of successful responses, the vertical component of the speed at the window was determined, followed by

$$\text{Speed} = \sqrt{(3 \cdot 523 \dots)^2 + (23 \cos 18^\circ)^2} = 22 \cdot 156 \dots > 21$$

This additional, unnecessary work still only earned the final B1 mark.

Q.9 This question was well done by the majority of candidates, particularly those who considered a clearly labelled force diagram. Many candidates were less successful in securing the final mark in part (c), as they were often unable to provide an articulate reason to support their answer.

Q.10 Efforts were very disappointing in this question. The attempt rate, of over 91 %, was strong, but the facility factor was very low at 28.5. The main issue was establishing the correct underlying differential equation for part (a)(i), since many candidates did not understand what was meant by 'inversely proportional'. The following was frequently seen,

$$\frac{dv}{dt} = \pm kv.$$

Supporting you

Useful contacts and links

Our friendly subject team are on hand to support you between 8.30am and 5.30pm, Monday to Friday.

Tel: 029 2240 4251

Email: mathematics@wjec.co.uk

Qualification webpage: <https://www.wjec.co.uk/qualifications/mathematics-a-as-level/>

See other useful contacts here: [Useful Contacts | WJEC](#)

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WJEC
245 Western Avenue
Cardiff CF5 2YX
Tel No 029 2026 5000
Fax 029 2057 5994
E-mail: exams@wjec.co.uk
website: www.wjec.co.uk

ⁱ *Please note that where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.*